Rotational Dynamics

Q. 1. *A uniform rod of length 1.2 meters and mass 3 kg is rotating about an axis perpendicular to its length and passing through one of its ends.*

(i) Calculate the moment of inertia of the rod about this axis.

(ii) If the rod is rotating with an angular velocity of 5 rad/s, determine the kinetic energy of the rod.

Solution:

Here, length, l = 1.3 m

Mass, m = 3 kg

(i) Moment of inertia, I = ?

We have for the axis at the end of the rod and normal to the rod,

$$I = \frac{Ml^2}{3} = \frac{3\times1.3^2}{3}$$

$$= 1.44 \text{ kg-m}^2$$

(ii) Angular velocity, ω = 5 $rads^{-1}$

$$\text{Therefore, KE} = \frac{1}{2} I\ \omega^2 = \frac{1}{2} \times 1.69 \times 5^2$$

$$= 18 \text{ J}$$

Hence the moment of inertia of the rod is 1.44 kg·m^2, and the kinetic energy of the rod is 18 J.

Q. 2. *A uniform* ***rod of*** *mass 2 kg and length 1.2 m is hinged at one end and is free to rotate in a vertical plane without friction. It is held horizontally and then released from rest.*

a) *What is the torque acting on the rod about the hinge just after it is released?*

b) *Calculate the angular acceleration of the rod at that instant.*

c) *What is the linear acceleration of the free end of the rod immediately after release?*

Solution:

Here, Mass of rod, M=2 kg

Length of rod, L=1.2 m

Acceleration due to gravity, g=9.8 m/s^2

Moment of inertia of rod about one end:

$$I = \frac{1}{3}ML^2$$

(a) Torque about the hinge just after release

Torque due to weight acts at the center of mass, which is at $\frac{L}{2}$ from the hinge.

$$\tau = Mg\cdot\frac{L}{2}$$

Substitute values:

$$\tau = 2 \times 9.8 \times \frac{1.2}{2} = 11.76 \text{ Nm}$$

b) Angular acceleration of the rod

Using the relation:

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$$

To calculate moment of inertia, we use $I = \frac{1}{3}ML^2 = \frac{1}{3} \times 2 \times 1.2^2 = 0.96 \text{ Kgm}^2$

Now $\alpha = \frac{\tau}{I} = \frac{11.76}{0.96} = 12.25 \text{ rads}^{-2}$

(c) Linear acceleration of the free end of the rod

The linear acceleration a at the end of the rod is related to angular acceleration α by:

$a = \alpha\cdot L \Rightarrow a = 12.25\times1.2 = 14.7 \text{ ms}^{-2}$.

Q.3. *The Earth has a radius of approximately 6371 km and completes one rotation every 24 hours. Calculate the tangential force needed to stop the Earth's rotation within 1 hour. (Mass of the Earth = 5.97×10^{24} kg)*

Solution: Here,

Radius of Earth $R = 6371$ km $= 6371\times10^3$ m

Rotational period $T = 24$ hours $= 24\times3600$ s

Stopping time $t_{stop} = 1$ hour $= 3600$ s

Mass of the Earth, $M = 5.97 \times 10^{24}$ kg

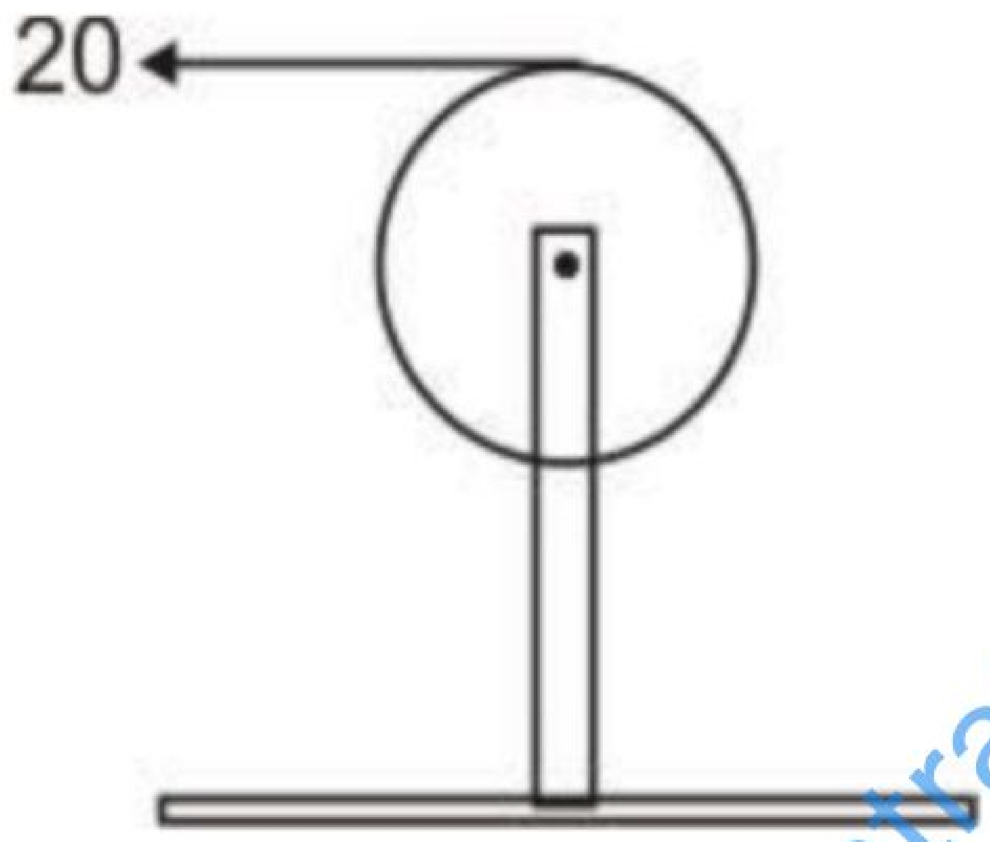
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times3600} \approx 7.27\times10^{-5} \text{ rad/s}$$

Angular acceleration, $\alpha = \frac{\omega_0 - \omega}{t_{stop}} = \frac{7.27\times10^{-5}}{3600}$ s $= 2.02 \times 10^{-8}$ rads^{-2}

The tangential acceleration, $a = R.\ \alpha = 6371\times10^3 \times 2.02 \times 10^{-8} = 0.1286$ ms^{-2}

Now tangential force, $F = Ma = 5.97 \times 10^{24} \times 0.1286 = 7.68 \times 10^{23}$ N

Q.4. *string is wrapped around the rim of a wheel of moment of inertia 0.1kgm² and of radius 20cm. The wheel is free to and free to rotate about its axis and initially it is at rest. The string is now pulled by a force of 20N. What is the angular velocity of the string after 2sec.?*



Solution: We know that, $\tau = F \times r$…. (1.11)

Also, $\tau = I \cdot \alpha$ (1.2)

From equation (1) and (2)

$F \times r = I \times \alpha$

$20 \times 0.2 = 0.1 \times \alpha$

$\alpha = 40$ rads^{-2}

Again, $\omega = \omega_o + \alpha t$

or, $\omega = 0 + 40 \times 2$

$\omega = 80$ rads^{-1}.

Q.5. An aircraft engine delivers a power of 200 kW to the propeller. The propeller rotates at a constant frequency of 1500 rpm. (a) Calculate the torque exerted by the engine on the propeller. (b) Determine the work done in one revolution of the propeller.

Solution: Here, power, $P = 200\ Kw = 2 \times 10^5\ W$

Frequency, $f = 1500\ rpm = \frac{1500}{60} = 25\ revs^{-1}$

$\Rightarrow \omega = 2\pi f = 2\pi \times 25 = 50\pi\ rad\ s^{-1}$

(a) Torque, $\tau = ?$

We have, $P = \tau\omega$

$$\Rightarrow \tau = \frac{P}{\omega} = \frac{2 \times 10^5}{50\pi} = 1272.72\ Nm$$

(b) Work, W = ?

We have, $W = \tau\theta = \frac{2 \times 10^5}{50\pi} 2\pi = 8000\ J$

Periodic Motion

Q.6. A particle performs Simple Harmonic Motion (SHM) with a period of T= 2 seconds. The amplitude of the motion is 6 cm. At t = 0, the particle is at its maximum displacement (i.e., at amplitude) and moving towards the equilibrium position.

(a) Find the angular frequency of the motion.
(b) Calculate the velocity and acceleration of the particle when its displacement is 4 cm.
(c) Determine the maximum speed and maximum acceleration of the particle.

Solution:
a) We know that the angular frequency ω is related to the period T by the formula:
$\omega = \frac{2\pi}{T}$
Given that T = 2 seconds, we can calculate ω:
$\omega = \frac{2\pi}{T} = \pi\ rads^{-1}$
So, the angular frequency = $\pi\ rads^{-1}$

(b) Calculate the velocity and acceleration when the displacement is y= 4 cm:

The displacement equation is given by:
$y = r\sin\omega t$
The velocity v is the time derivative of displacement:
$v = d/dt[r\sin\omega t] = r\omega\cos\omega t$
The acceleration a is the time derivative of velocity:
$A = d/dt[r\omega\cos\omega t] = -r\omega^2\sin\omega t = -\omega^2 y$
We now need to find the velocity and acceleration when the displacement is y= 4 cm.
Given:
y = 4 cm.
From the displacement equation $y = r\sin\omega t$, we have:
$4 = 6\sin\pi t$
Solving for $\sin\pi t$

$\sin\pi t = \frac{2}{3}$

Now we can use this to find the velocity and acceleration.

Velocity, $v = r\omega\cos\omega t$

Now, we calculate the velocity:

$v(t) = 6\times\pi\times\frac{\sqrt{5}}{3} = 2\sqrt{5}\pi$ cm

$a = -r\omega^2\sin\omega t$

Substitute the known values:

$a = -6\times\pi^2\times\frac{2}{3} = -4\pi^2$ cms^{-2}

(c) Determine the maximum speed and maximum acceleration:

Maximum speed occurs when $\sin\omega t = 1$ i.e., when the particle is at the equilibrium position).

At that point:

$V_{max} = r\omega = 6\times\pi = 6\pi$ cms^{-1}

Maximum acceleration occurs when $\sin\omega t = 1$

At that point:

$a_{max} = r\omega^2 = 6\times\pi^2$ cms^{-2}

Fluid Mechanics

Q.7. A U-shaped tube, open at both ends and filled with water, has oil (which does not mix with water) poured into one side. This causes the water level to rise by 65 cm on the other side, while the oil level stands 10 cm higher than the water level. Given this setup, the density of the oil is:

a. 928 kgm^{-2} b. 650 kgm^{-2} c. 425 kgm^{-2} d. 800 kgm^{-2}

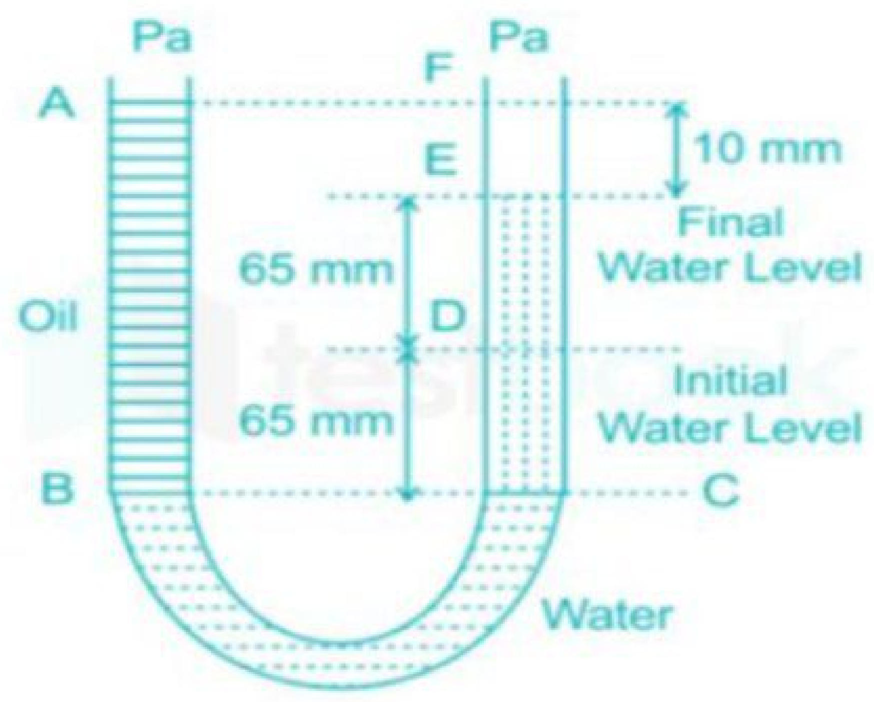


Fig : 3.11

Solution: Using the relation, $P = \rho gh$, we have $\rho_0 g h_0 = \rho_w g h_w$

Or, ρ_0 =w hwh0 = 1000×130140 = 928kgm^{-2}

Q.8 *A fluid of viscosity 0.001 Pa·s flows through a narrow pipe of length 0.5 m and radius 1 mm. If the pressure difference across the pipe is 2000 Pa, calculate the volume flow rate.Now, suppose the radius is doubled while all other conditions remain the same. By what factor does the flow rate change? Calculate the new flow rate.*

Solution: Here in the first case, we use the flow of the volume rate, $V = \frac{\pi p r^4}{8\eta l}$.

With p = 2000 Pa, r = 1 mm = 10^{-3} m, η = 0.001 Pa, l = 0.5 m; $V = \frac{\pi\times2000\times10^{-3\times4}}{8\times0.001\times0.5} = \mathbf{1.57\times10^{-6}\ m^3/s}$.

For the second part when the radius is doubled, r - 2×10^{-3} m, then from the above's relation,

$$V' = \frac{\pi\times2000\times2\times10^{-3\times4}}{8\times0.001\times0.5} = 2.51\times10^{-5}\ m^3/s$$

And the factor of flow rate = $\frac{V'}{V}$ = 16

Q.9. *You are designing a sedimentation tank for separating plastic particles from a fluid. You want the plastic spheres (density =1050 kg/m^3) to fall with a terminal velocity of at least 2.0 mm/s in a fluid of viscosity 1.2 Nsm^{-2} and density 1000 kg/m^3. What should be the minimum radius of the plastic spheres to ensure this condition?*

Solution:

Using the Stoke's formula $\eta = \frac{2r^2(\rho-\sigma)g}{9v} \Rightarrow r = \sqrt{\frac{9\eta v}{2(\rho-\sigma)g}}$

With η = 1.2Nsm^{-2} , v = 2 mm/s = 2×10^{-3} m/s, ρ = 1050 kgm^{-3}, σ = 1000 kgm^{-3};

$$Rr = \sqrt{\frac{9\times1.2\times2\times10^{-3}}{2(1050-1000)\times9.8}} = 4.69\ mm$$

Q.10. *Blood flows through an artery at a speed of 0.4ms^{-1} in a region where the artery has a radius of 3mm. Further along, the artery narrows due to a blockage, reducing its radius to 1.5mm.*

i) Calculate the speed of blood flow in the constricted region.

ii) By what factor does the velocity increase due to the narrowing?

Solution: Here, velocity in wider section: $v_1 = 0.4ms^{-1}$

Radius in wider section: r_1=3 mm=0.003 m

Radius in narrower section: r_2=1.5 mm = 0.0015m.

$v_1 = 0.4\ ms^{-1}$, $A_1 = \pi r_1^2 = \pi\times(3\times10^{-3})^2 = 9\pi\times10^{-6}\ m^2$

$A_2 = \pi r_2^2 = \pi\times(1.5\times10^{-3})^2 = 2.25\pi\times10^{-6} m^2$

i) Speed of the constricted region, v_2 = ?

We have, $A_2v_2 = A_1v_1 \Rightarrow v_2 = \frac{A_1v_1}{A_2} = \frac{9\pi\times10^{-6}\times0.4}{2.25\pi\times10^{-6}} = 1.6\ ms^{-1}$

ii) Here $\frac{v_2}{v_1} = \frac{1.6}{0.4}$ = 4 is the factor that the velocity increases.

Q.11. The volume of steam produced by 1 g of water at 100°C s 1650 cm^3. Calculate the change in internal energy during the change of state. Given J = 4.2×10^7 erg cal^{-1}, g = 981 cm s^{-2}. Latent heat of steam = 540 cal g^{-1}.

Solution:

Given, J = 4.2×10^7 erg cal^{-1}

Latent heat of steam = 540 cal g^{-1} Mass of water = 1 g

Temperature of water = 100°C Initial volume (V_1) = 1 cm^3

Final volume (V_2) = 1650 cm^3

∴ Change in volume (dV) = $V_2 - V_1 = 1650 - 1 = 1649$ cm^3

When 1 g of water at 100°C is changed to steam at 100°C, temperature will remain constant. Therefore, heated supplied

dQ = latent heat of steam = 540 cal= $540 \times 4.2 \times 10^7$ erg

Also, $dQ = dU + PdV$

$\therefore\ dU = dQ - PdV$

Taking P = 1 atmosphere = $76 \times 13.6 \times 981$ dyne cm^{-2}

We have, $dU = 540 \times 4.2 \times 10^7 - 76 \times 13.6 \times 981 \times 1649$

$= 22.68 \times 10^9 - 1.67 \times 10^9$

$= 2.1 \times 10^{10}$ erg.

Q.12. A mass of air occupying initially a volume $2 \times 10^{-3} m^3$ at a pressure of 760mm of mercury and a temperature of 20°C is expanded adiabatically and reversibly to twice its volume, and then compressed isothermally and reversibly to a volume of $3 \times 10^{-3} m^3$. Find the final temperature and pressure (γ for air = 1.4)

Solution:

Given, Initial volume (V_1) = $2 \times 10^{-3} m^3$

Initial pressure (P_1) = 760 mm of Hg

Initial temperature (T_1) = 293K

Final volume (V_2) = $2V_1$

We have for adiabatically process

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\therefore\ P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 = \left(\frac{V_1}{2V_1}\right)^{1.4} \times 760$$

= 288 mm of Hg

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\therefore\ T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} T_1 = \left(\frac{V_1}{2V_1}\right)^{1.4-1} \times 293 = 222K$$

$V_3 = 3 \times 10^{-3} m^3$, P_3 =?

As the process is isothermally, so the temperature

T_3 = 222°k

$P_2V_2 = P_3V_3$

$$\therefore\ P_3 = \frac{V_2P_2}{V_3} = \frac{2 \times 2 \times 10^{-3}}{3 \times 10^{-3}} = 384 \text{ mm of Hg}$$

Q.13. Air initially at 27°Cand 750 mm of Hg pressure is compressed isothermally until its volume is halved. It is than expanded adiabatically until its original volume is recorded. Assuming the change to be reversible, find the final temperature and pressure (γ for air = 1.4).

Solution:

Given, Initial temperature (T_1) = 300K

Initial Pressure (P_1) = 750 mm of Hg

Let original volume be V m³

(i) Isothermally compression, $V_2 = \frac{V}{2}$

$$P_1V_1 = P_2V_2$$

$$\therefore \quad P_2 = \frac{P_1V_1}{V_2} = \frac{2V \times 750}{V} = 1500 \text{ mm of Hg}$$

(ii) Adiabatically expansion, $V_3 = V$

$$P_3V_3^{\gamma} = P_2V_2^{\gamma}$$

$$\therefore \quad P_3 = \left(\frac{V_2}{V_3}\right)^{\gamma}$$

$$P_2 = \left(\frac{V}{2V}\right)^{1.4} \times 1500 = 568.4 \text{ mm of Hg}$$

$$T_3V_3^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\therefore \quad T_3 = \left(\frac{V_2}{V_3}\right)^{\gamma-1} T_2 = \left(\frac{V}{2V}\right)^{1.4-1} \times 300 = 227.36\text{K}$$

2nd Law of Thermodynamics

Q.14. A Carnot's engine has the same efficiency

(i) Between 100 K and 500 K and (ii) Between T K and 900 K

Calculate the temperature TK of the sink

Solution:

(i) Given,

Temperature of source (T_1) = 500 K

Temperature of sink (T_2) = 100 K

Efficiency (η)

We have, efficiency (η) $= 1 - \frac{T_2}{T_1} = 1 = \frac{100}{500} = 10.2 = 0.8$

(ii) Temperature of source (T_1) = 900 K

Temperature of sink (T) =?

Since, $\eta = 1 - \frac{T_2}{T_1}$

$\therefore \quad 0.8 = 1 - \frac{T}{900}$

or $\frac{T}{900} = 1 - 0.8 = 0.2$

or T = 180 K

Hence, temperature of sink is 180 K

Q.15. Carnot engine absorbs 100J of heat from a reservoir at 127°C and rejects 600 J of heat during each cycle. Calculate

(i) The efficiency of the engine

(ii) The temperature of the sink

(iii) The amount of the useful work done during each cycle.

Solution:

Given,

Heat taken from source (Q_1) = 1000 J

Heat reject to sink (Q_2) = 600 J

Temperature of source (T_1) = 127 + 273 = 400 K

Efficiency (η) =?

Temperature of sink (T_2) = ?

Amount of useful work done during each cycle (W) =?

(i) Now, we have efficiency (η) $= 1 - \frac{Q_2}{Q_1} = 1 - \frac{600}{100} = 1 - 0.6 = 0.4$

$\therefore$ %η $= 0.4 \times 100 = 40\%$

(ii) Now, $\frac{Q_2}{Q_1} = \frac{T_1}{T_2}$

$\therefore \quad T_2 = T_1 \frac{Q_2}{Q_1} = 400 \times \frac{600}{1000}$

$= 240K = 240 - 273 = -33°C$

(iii) Amount of useful work done during each cycle,

$W = Q_1 - Q_2 = 1000 - 600 = 400$ J

Hence, the efficiency, temperature of sink and amount of useful work done during each cycle
are 40%, – 33°C and 400 J respectively.

Q.16. A reversible engine converts one sixth of heat input into work. When the temperature of the sink is reduced by 62°C its efficiency is doubled. Find the temperature of the source and the sink

Solution: Given,

$$\text{Efficiency } (\eta) = \frac{W}{Q_1} = \frac{1}{6}$$

Also, $\eta = 1 - \frac{T_2}{T_1}$

$\therefore \quad \frac{1}{6} = 1 - \frac{T_2}{T_1}$

or $\quad \frac{T_2}{T_1} = \frac{5}{6}$

or $\quad T_1 = 1.2\, T_2$ .....(i)

Now, when temperature of sink is decreased by 62°C *i.e.*, 62 K, efficiency becomes double *i.e.* when

$T'_2 = T_2 - 62, \qquad \eta' = \frac{1}{6} \times 2 = \frac{1}{3}$

Now, $\eta' = 1 - \frac{T'_2}{T_1}$

or, $\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$

or, $\frac{T_2 - 62}{T_1} = 1 - \frac{1}{3} = \frac{2}{3}$

or, $T_1 = \frac{3}{2}(T_2 - 62) = 1.5\,(T_2 - 62)$ .....(ii)

From (i) and (ii), we get

$1.5\,(T_2 - 62) = 1.2\, T_2$

or, $0.3\, T_2 = 62 \times 1.5$

or, $T_2 = \frac{62 \times 1.5}{0.3} = 310$ K

Using equation (i), we get

$T_1 = 1.2 \times 310 = 372$ K

Hence, the temperature of the source and the sink are 372 K and 310 K.

Q.17. A plane progressive wave is represented by the equation $y = 0.1 \sin (200\pi t - \frac{20\pi x}{17})$ where y is the displacement in mm, t is in sec, an x is the distance from a fixed origin O in meter(m). Find (i) the frequency of the wave, (ii) its wavelength, (iii) its speed, (iv) the phase difference in radians between a point 0.25m from O and a point 1.10 m from O.

Solution: Given,

$$y = 0.1 \sin \left(200\ \pi t - \frac{20\ \pi x}{17}\right)$$

Compare this eqn. with $y = a \sin (\omega t - kx)$

We get,

Amplitude (a) $= 0.1$ mm $= 10^{-4}$ m

Angularvelocity (ω) $= 200\ \pi = 2\pi f$

Wave vector (k) $= \frac{2\pi}{\lambda} = \frac{20\pi}{17}\ m^{-1}$

i. Frequency(f)$= \frac{\omega}{2\pi} = 100$ Hz

ii. Wavelength (λ)$= \frac{2\pi}{k} = \frac{2\pi}{20\pi} \times 17 = 1.7$ m

iii. Speed (v)$= f\lambda = 170\ ms^{1}$

iv. Path difference (x) $= (1.10 - 0.25)$ m $= 0.85$

Phase difference (ϕ) $= \frac{2\pi}{\lambda} x = \frac{2\pi}{1.7} \times 0.85 = \pi$

Q.18. Stationary waves are set up by the superposition of two waves given by $y_1 = 0.05 \sin (5\pi t - x)$ and $y_2 = 0.05 \sin (5\pi t + x)$ where x and y are in meters and t in sec. Find the amplitude of a particle situated at the distance, x = 1m.

Solution:

According to the principle of superposition the resultant displacement y is given by

$$y = y_1 + y_2 = 0.05 \sin (5\pi t - x) + 0.05 \sin (5\pi t + x)$$

$$= 0.05 \times 2 \sin \frac{5\pi t - x + 5\pi t + x}{2} \cos \frac{5\pi t + x - 5\pi t + x}{2}$$

$$y = 0.1 \cos x \sin 5\pi t$$

$\therefore$ Amplitude (a) $= 0.1 \cos x$ at $x = 1$ m

Amplitude (a) $= 0.1 \cos 1 = 0.1 \cos \frac{180^0}{\pi}$ $\left[\because \phi = \frac{2\pi}{\lambda} x\right]$

$= 0.1 \times 0.5402 = 0.054$ cm

Q.19. Find the temperature at which the velocity of sound in air is double the velocity of sound in air at 0°C.

Solution: Given,

Let v_0 be the velocity of sound at 0°C anT°C be the temperature at which the velocity of sound will be double the velocity of sound in air at 0^EC, i.e. $v_t = 2v_0$.

$$\therefore \quad \frac{v_t}{v_0} = \sqrt{\frac{273 + T}{273}}$$

or, $$\sqrt{\frac{2v_0}{v_0}} = \sqrt{\frac{273 + T}{273}}$$

or, $$2 = \sqrt{\frac{273 + T}{273}}$$

Squaring both sides,

$$4 = \frac{273 + T}{273}$$

or $4 \times 273 = 273 + T$

or, $T = (4 \times 273 - 273) = 3 \times 273 = 819°C$

Q.20. If a detonator is exploded on a railway line, an observer standing on the rail 2 km away hears two reports. What is the time interval between these two reports? [Young's modulus for steel = 2.0×10^{11} Nm^{-2}, Density of steel = 8.0×10^3 kgm^{-3}, density or air = 1.4 kgm^{-3}, Ratio of molar heat capacities of air = 1.40, Atmospheric pressure = 10^5 Nm^{-2}]

Solution: Given,

Modulus of steel (Y) = 2.0×10^{11} Nm^{-2}

Density of steel (ρ_{Steel}) = 8.0×10^3 kgm^{-3}

Density of air (ρ_{air}) = $1.4 kgm^{-3}$

Pressure (P) = $10^5 Nm^{-2}$

Ratio of heat capacity (γ) = 1.40

Velocity of sound in air (v_a)

$$= \sqrt{\frac{\gamma P}{\rho_{air}}}$$

$$= \sqrt{\frac{1.40 \times 105}{1.4}}$$

= 316.23m/s

$\therefore$ Time taken by sound to travel 2km trough air $t_a = \frac{2000}{316.23} = 6.32$secs

Velocity of sound through steel rails (v_s)

$$= \sqrt{\frac{Y}{\rho_{steel}}} = \sqrt{\frac{2.0 \times 10^{11}}{8.0 \times 10^3}} = \sqrt{\frac{10^8}{4}} = \frac{10^4}{2} = 5000 ms^{-1}$$

$\therefore$ Time taken by sound to travel through 2km of steel

$$t_s = \frac{2000}{v_s} = \frac{2000}{5000} = \frac{2}{5} = 0.4 secs.$$

$\therefore$ Time interval between two reports

= (6.23 - 0.4) sec = 5.92secs.

Hence, the time between two reports is 5.92secs.

Q.21. **The interval between the flash lightening and the sound of thunder is 1 second, when the temperature is 10°C. How far is the storm if the velocity of sound in air at 0°C is 330 m/sec?**

Solution: Given,

Time interval (t) = 1 sec

Temp. (T) = 10°C = 283K

Velocity of sound at 0°C (V_0) = 330ms^{-1}

Distance of storm =?

Now,

$$\frac{v_1}{v_0} = \sqrt{\frac{T_1}{T_0}}$$

or, $$\frac{v_1}{330} = \sqrt{\frac{283}{273}}$$

or, v_1 = 336 ms^{-1}

Now, distance covered by the sound with the velocity of 336ms^{-1} in 1 sec. is d then

$$1 = \left(\frac{d}{v} - \frac{d}{c}\right) = d\left(\frac{1}{336} - \frac{1}{3\times10^8}\right)$$

d = 336m.

Hence, storm is 236m far.

Waves in Pipe and Strings

Q.22. ***Find the ratio of the length of a closed pipe to that of an open pipe in order that the second overtone of the former is in unison with the fourth overtone of the latter.***

Solution:

Let l_1 be the length of the closed pipe and l_2 be that of an open pipe. For the second overtone of the closed pipe, length (l_1) = $\frac{5\lambda}{4}$ or $\lambda = \frac{4l_1}{5}$

For the fourth overtone of the open pipe

$$l_2 = \frac{5\lambda'}{2} \qquad \text{or } \lambda' = \frac{2l_2}{5}$$

∴ The frequency of vibrations is the same in both cases.

$$v = f\lambda = f\lambda'$$

or, $\lambda = \lambda'$

$$\frac{4l_1}{5} = \frac{2l_2}{5}$$

or, $l_1 : l_2 = 1:2$

Hence, ratio of the length of a closed pipe is 1:2.

Q.23. **Compare the fundamental frequencies of open and closed organ pipes of the same length.**

Solution:

For the open pipe, the fundamental frequency f_0, is given by

or, $f_0 = \frac{v}{\lambda_0} = \frac{v}{2l}$ .....(i)

For the closed pipe, the fundamental frequency f_0' given by

or, $f_0' = \frac{v}{\lambda_0'} = \frac{v}{4l}$ .....(ii)

Dividing (i) by (ii)

$$\frac{f_0}{f_0'} = \frac{v}{2l} \times \frac{4l}{v} = 2$$

Q.24. **A steel wire 1m long who's mass is 1.0 g is under a tension of 400N and is tied down at both ends.**

a. What is the wavelength of its fundamental mode of vibration?

b. What is the frequency of this mode?

Solution: Given,

Length (l) = 1m

Tension (T) = 400N

Mass per unit length (μ) = $\frac{1.0}{1}$ g/m = 10^{-3} kg/m

For fundamental mode of vibration

$$\frac{\lambda}{2} = l \text{ or } \lambda = 2l = 2 \times 1 = 2\text{m}$$

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 1}\sqrt{\frac{400}{10^{-3}}} = 316\text{Hz}$$

Hence, the frequency of this mode is 316Hz.

Q.25. **A piano string 1.5 m long is made of steel of density 7800 kgm^{-3} and Young's modulus 2×10^{11} Nm^{-2}, it is maintaining at a tension which produced an elastic strain of 1% in the string. Calculate the frequency of transverse vibration of the string when it is vibrating in second mode of vibration.**

Solution: Given,

Length of the string (l) = 1.5 m

Density of the string (ρ) = 7800 kgm^{-3}

Young's modulus (Y) = 2×10^{11}Nm^{-2}

Strain= 1% = 0.01

Frequency(f) =?

We know,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{Tl}{m}} = \sqrt{\frac{Tl}{V.\rho}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}}$$

Again Young's modulus of elasticity (Y) = $\frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\text{Strain}} \rightarrow \frac{T}{A}$ = Y strain

Then,

$$f = \frac{1}{2l}\sqrt{\frac{\text{Y strain}}{\rho}} = \frac{1}{2 \times 1.5}\sqrt{\frac{2 \times 10^{11} \times 0.01}{7800}} = 168.81 \text{ Hz.}$$

Hence, the frequency of fundamental mode of vibration is 168.81 Hz.

Q.26 *A note of frequency 300Hz has an intensity of 1 μW/m². What is the amplitude of air oscillation* **by the sound? [Density of air = 0.029kg/m³, Bulk modulus = 1.42 × 10⁵ N/m²]**

Solution: Given,

Intensity (I) = 1μW/m² = 1×10^{-6}W/m²

Density (ρ) = 0.029kg/m²

Frequency (*f*) = 300Hz

Bulk modulus of elasticity (B) = 1.42×10^5N/m²

We have,

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

$$\text{Amplitude, } A^2 = \frac{2I}{\sqrt{\rho B}\omega^2} = \frac{2I}{\sqrt{\rho B} \times 4\pi^2 f^2} = \frac{I}{\sqrt{\rho B} \times 2\pi^2 f^2} = \frac{2I}{\sqrt{\rho B}\, 2\pi^2 f^2}$$

$$= \frac{10^{-6}}{\sqrt{0.029 \times 1.42 \times 10^5} \times 2p^2 \times (300)^2} = 0.8 \times 10^{-14} \text{ m}$$

∴ A = 0.89×10^{-7}m

Hence, the amplitude of oscillation is 0.89×10^{-7}m.

Q.27. ***A loudspeaker produces a sound intensity level of 8 decibels above a certain reference level at a point 40m from it. Find the intensity level at a point 30m from the loudspeaker.***

Solution:

Let I_1 and I_2 be the sound intensities at point 40m and 30m from the loudspeakers.

$$\therefore \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} = \left(\frac{40}{30}\right)^2$$

The intensity level at the second point is higher than that at the 1st point by 'b' decibels where

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$= 10 \log_{10}\left[\frac{40}{30}\right]^2$$

Hence, intensity level at the second point = 8 + 2.5 = 10.5 decibels.

Q.28. **A source of sound generates sound waves which travel with a speed of 340 m/s. The frequency of the source is 500 Hz. Find the frequency of the sound heard if**

a) The source is moving towards the stationary observer with a speed of 30 m/s

b) The observer is moving towards the stationary source with a speed of 30 m/s

c) Both source and observer move with a speed of 30 m/s and approach one another.

Solution: Given,

Speed of sound (v) = 340 m/s

Frequency of source (*f*) = 500 Hz

a) Speed of source (u_s) = 30 m/s

Frequency of sound observed by stationary observer is given by,

$$f' = \left(\frac{v}{v - u_s}\right)f = \left(\frac{340}{340 - 30}\right)500$$

∴ f' = 548.38 Hz.

b) Speed of the observer (u_o) = 30 m/s

Frequency heard by the observer is given by,

$f' = \left(\frac{v + u_0}{v}\right) f = \left(\frac{340 + 30}{340}\right) 500$

$\therefore f' = 544.11$ Hz

c) Speed of observer (u_o) = 30 m/s

Speed of the source (u_s) = 30 m/s

Frequency heard by observer when source and observer approach each other is given by,

$$f^1 = \left(\frac{v + u_0}{v - u_s}\right) f$$

$$= \left(\frac{340 + 30}{340 - 30}\right) 500$$

$\therefore \quad f' = 596.4$ Hz

Hence, the frequency heard by the observer are 548.38 Hz, 544.11 Hz and 596.4 Hz.

Q.29. A stationary motion detector sends sounds waves of 150 KHz towards a truck approaching at a speed of 120 km/hour. What is the frequency of ware reflected back to detector? [velocity of sound in air = 340 m/s]

Solution: Given,

Frequency of source (f) = 150000 Hz

Speed of truck (u_s) $= \frac{120 \times 100}{60 \times 60} = \frac{200}{6} = 33.33$ m/s

Velocity of sound (v) = 340 m/s

In first case: motion detector act as a source and truck as observer.

Now,

Apparent frequency (f) =?

We have, $f = \frac{v + u_o}{v} \times f$

[∵ truck is presented us source since the sound decked by detritus is reflected by truck.]

$\because \; f = \frac{340 + 33.33}{340} \times 150 = 164.7$ KHz

In second case: motion detector at as a observer and truck as a source, then

$$f'' = \frac{v}{v - u_s} \times f = \frac{340}{340 - 33.33} \times 164.7 = 182.6 \text{ KHz}$$

Hence, the frequency detect by motion detector is 182.6 KHz

Interference

Q.30. *In a double-slit experiment with light of wavelength 500 nm and screen distance 2.0 m, the slit separations are 0.1 mm and 0.05 mm in two cases. Find the ratio of the fringe widths formed in the two cases.*

Solution:

Here, wavelength, $\lambda = 500$ nm $= 5.0 \times 10^{-7}$m,

The slit-screen distance, D = 2.0 m,

Slit separation, $d_1 = 0.1$ mm $= 1.0 \times 10^{-4}$ m and $d_2 = 0.05$ mm $= 5.0 \times 10^{-5}$ m

From $\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \frac{1}{d} \Rightarrow \frac{\beta_1}{\beta_2} = \frac{d_2}{d_1} = \frac{0.05}{0.1} = \frac{1}{2}$

Q.31. *In a double-slit experiment with light of wavelength λ=450nm, the distance between the slits is d=1.0 mm and the screen is placed D=5.0 m away. Find the angle for the first order maximum.*

Solution:

Here, wavelength, $\lambda = 450$ nm $= 4.5 \times 10^{-8}$m

The slit-screen distance, D = 5.0 m,

Slit separation, $d_1 = 1.0$ mm $= 1.0 \times 10^{-3}$ m

We have, path difference, $d\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

For the first order, n = 1 and hence $\sin\theta_1 = \frac{\lambda}{d} \Rightarrow \theta_1 = \sin^{-1}(\frac{\lambda}{d})$

$= \sin^{-1}\frac{4.5\times10^{-8}}{10^{-3}} = 0.0257°$

Diffraction

Q.32. *A diffraction grating with 5000 lines per centimeter is used to diffract light of wavelength 600 nm (nanometers). The grating is placed perpendicular to a monochromatic light source, and the diffraction pattern is observed on a screen placed 2 meters away from the grating.*
i) Determine the angle of diffraction for the first-order (n = 1) and second-order (m = 2) maxima for the given wavelength of light.
ii) What is the minimum distance between the first-order and second-order maxima on the screen?

Solution: Here, Given:

Grating lines per centimeter = 5000 ⇒ $d = \frac{1}{5000/cm} = \frac{1}{5000/10^{-2}}$ m = 2×10^{-6} m

Wavelength of light (λ) = 600 nm = 600×10^{-9} m

Distance from the grating to the screen (D) = 2 m

i) For the first order diffraction, $d\sin\theta_1 = n\lambda \Rightarrow \theta_1 = \sin^{-1}(\frac{\lambda}{d})$, n = 1

$= \sin^{-1}(\frac{6\times10^{-9}}{2\times10^{-6}})$

$= \sin^{-1}(0.003) = 0.172^0$

For the 2nd order diffraction, $d\sin\theta_1 = n\lambda \Rightarrow \theta_2 = \sin^{-1}(\frac{2\lambda}{d})$, n = 2

$= \sin^{-1}(\frac{2\times6\times10^{-9}}{2\times10^{-6}})$

$= \sin^{-1}(0.006) = 0.344^0$

(ii) For the 1st order, $\tan\theta_1 = \frac{x_1}{D} \Rightarrow x_1 = D\tan\theta_1 = 2\times\tan0.00172 = 0.06$ cm

And for the 2nd order, $x_2 = D\tan\theta_2 = 2\times\tan0.0344 = 0.12$ cm

Hence the required distance = $\Delta x = x_2 - x_1 = 0.12 - 0.06 = 0.06$ cm.

Q.33. *A diffraction grating is used to observe the diffraction pattern of light with a wavelength of 550 nm. The first-order diffraction maximum is observed at an angle of 20°. The grating is placed perpendicular to a monochromatic light source, and the diffraction pattern is projected onto a screen. Calculate the number of rulings (lines per meter) on the diffraction grating.*

Solution: Here, Wavelength of light, λ=550 nm=550×10^{-9} m

Wavelength of light, λ=550 nm=550×10^{-9} m

Angle of diffraction for the first-order maximum, $\theta_1 = 20^0$

We have for the first order maximum, $d = \frac{\lambda}{\sin\theta} = \frac{550\times10^{-19}}{\sin20} = 1.61\times10^{-6}$ m

Now the number of lines per meter, $\frac{1}{d} = \frac{1}{1.61\times10^{-6}} = 6.21\times 10^5$ lines/m.

Polarization

Q.34. *A beam of light is incident at polarizing angle on a piece of a transparent material of refractive index 1.62. What is the angle of refraction for the transmitted beam?*

Solution: Here,

Refractive index, $\mu = 1.62$

We have $\tan\theta_p = \mu \Rightarrow \theta_p = \tan^{-1}\mu$

$= \tan^{-1}1.62 = 58.31^0$

We have $\theta_p + r = 90^0$

$\therefore r = 90^0 - 58.31^0 = 31.69^0$

Q.35. *Calculate the polarizing angle for a light ray travelling from water of refractive index 1.33 to glass of refractive index 1.53.*

Solution: Here, $\mu_w = 1.33$, $\mu_g = 1.53$

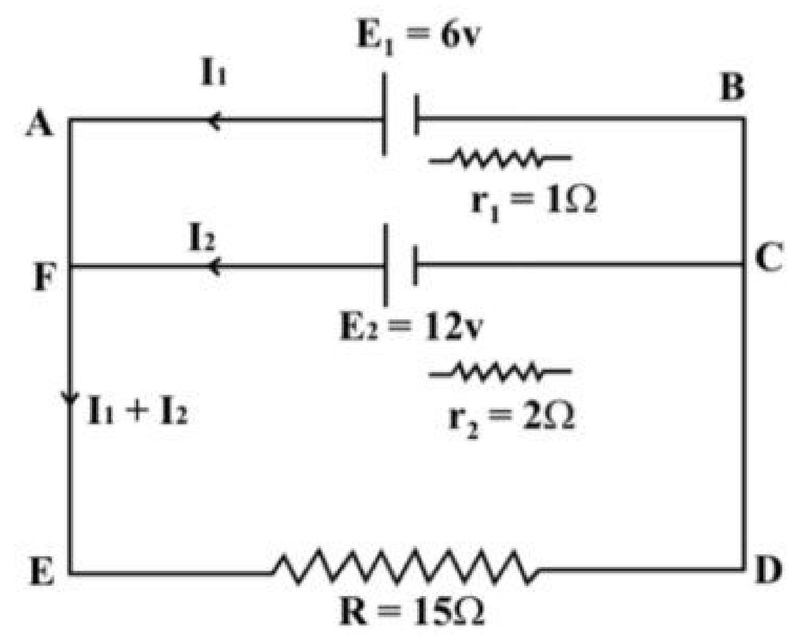
Polarizing angle, $\theta_p = ?$

We have, $\tan\theta_p = {}_w\mu_g \Rightarrow \theta_p = \tan^{-1}({}_w\mu_g) = \tan^{-1}\frac{\mu_g}{\mu_w} = \tan^{-1}\frac{1.53}{1.33} = 49^0$

Electrical Circuits

Q.36. Two batteries of e.m.f. 6V and 12V and internal resistance 1Ω and 2Ω respectively are connected in parallel with a resistance of 15Ω. Find the current through each branch of the circuit and potential difference across the resistance.

Solution:



Let current due to cells of e.m.f. E_1 and E_2 be I_1 and I_2 as shown in figure. Then, according to Kirchhoff's first law, the current through R = 15Ω will be $I_1 + I_2$.

Applying second Kirchhoff's law to closed part BAEDB, we have

$E_1 = I_1r_1 + (I_1 + I_2) R$

or, $6 = I_1 \times 1 + (I_1 + I_2)15$

or, $16I_1 + 15I_2 = 6$ ....(i)

Applying second Kirchhoff's law to closed part CFEDC, we have

$E_2 = I_2r_2 + (I_1 + I_2) R$

or, $12 = I_2 \times 2 + (I_1 + I_2)15$

or, $15I_1 + 17I_2 = 12$ ......(ii)

Solving equations (i) and (ii), we obtain

$I_1 = -1.66A$ and $I_2 = 2.17A$

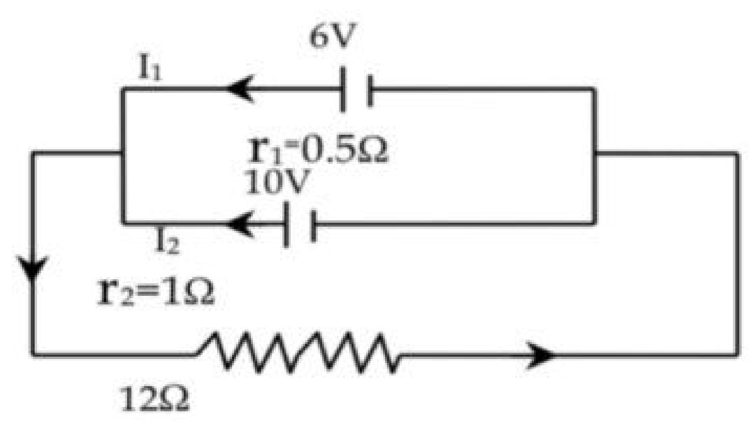
Since I_1 is negative, the current due to cell E_1 will flow in the direction opposite to that of I_1 shown in the figure.

Now, $I_1 + I_2 = -1.66 + 2.17 = 0.51A$

$\therefore$ Potential difference across $R = (I_1 + I_2)R = 0.51 \times 15 = 7.65V$

Hence, the potential difference across is 7.65V.

Q.37. A battery of 6V and internal resistance 0.5Ω is joined in parallel with another battery of 10V and internal resistance 1Ω The combination sends a current through an external resistance of 12Ω find current through each wire.



Solution, Given,

E.m.f. of 1st battery (E_1) = 6V

Internal resistance of 1st battery (r_1) = 0.5Ω

E.m.f. of 2nd battery (E_2) = 10V

and internal resistance (r_2) = 1Ω

External resistance (R) = 12Ω

Using Kirchhoff's law in upper loop

$6V = 0.5I_1 + (I_1+I_2)12$

$6V = 12.5I_1 + 12I_2$ .... (i)

In lower loop

$10V = I_2+12(I_1+I_2)$

$10 = 13I_2+12I_1$............. (ii)

By solving eq^n (i) and (ii) we get

$162.5I_1 + 156I_2 = 78$

$\underline{144I_1 + 156\,I_2 = 120}$

$18.5I_1 = -42$

$I_1 = -2.77A$

Putting the value of I_1 in equation (i) we get

$-12.5 \times 2.27 + 12I_2 = 6$

$-28.375 + 12I_2 = 6$

$12I_2 = 34.375$

$I_2 = 2.86A.$

Hence, the current following through each resistance

is 2.77A and 2.86A.

Q.38. The resistance in the left gap of a meter bridge is 10Ω and the point is 40cm from the left end. Calculate the value of the unknown resistance.

Solution: Given,

Length (l) = 40cm

Resistance (R) = 10Ω

Now, we have

$$\frac{P}{Q} = \frac{X}{R}$$

$$X = R \times \frac{l}{100 - l}$$

$$= 10 \times \frac{40}{100 - 40} = 10 \times \frac{40}{60} = 6.66\Omega$$

Hence, the value of unknown resistance is 6.66Ω.

Q.39. Four resistance 5Ω, 50Ω, 6Ω, 60Ω are connected to the corresponding arms of the Wheatstone bridge. If a cell of e.m.f. 1.5V and negligible internal resistance is connected across the bridge, calculate the current in arms and the cell. When the bridge is in balanced position.

Solution:

Here, the resistance is connected as shown in figure below.

Now, $\frac{P}{Q} = \frac{R}{X}$

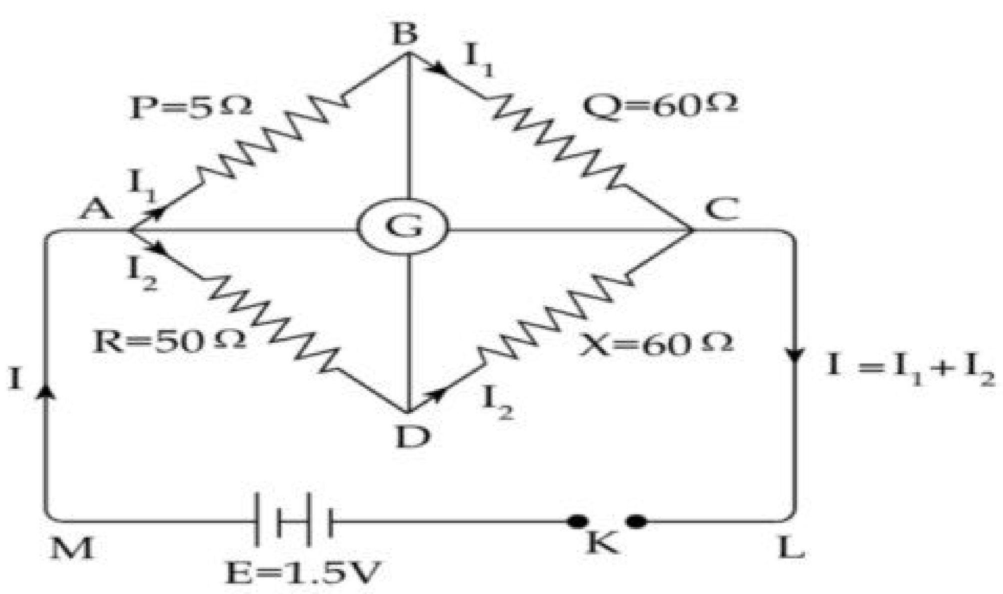
i.e. $\frac{5}{6} = \frac{50}{60}$

The bridge is in balanced position and hence no current flows through the galvanometer. Suppose the distribution of current is as shown. In the closed part ABDA from 2nd Kirchoff's law, we have,

$-I_1 \times 5 + I_2 \times 50 = 0$

or, $-I_1 + 10I_2 = 0$

or, $I_2 = \frac{I_1}{10}$



In the closed part ABCLMA, we have

$-5I_1 - 6I_1 - (I_1 + I_2) \times 0 = -1.5$

or, $11I_1 = 1.5$

or, $I_1 = 0.1363A$

$\therefore\ I_2 = \frac{I_1}{10} = \frac{0.1363}{10} = 0.01363A$

$\therefore\ I = I_1 + I_2 = 0.1363 + 0.01363$

$= 0.15\ A$

Q.40. P, Q, R and X are four coils of wire of 2, 2, 2 and 3Ω resistances respectively arranged to form a Wheatstone bridge. Calculate the value of resistance with which the coil X-must be shunted in order that the bridge may be balanced.

Solution:

For the bridge to be balanced, we have,

$$\frac{P}{Q} = \frac{X}{R} \text{ .... (i)}$$

If the X is shunted which means another resistance S must be connected in parallel with XS

From first equation

$$\frac{P}{Q} = \frac{X \| S}{R}$$

$$\text{or, } \frac{P}{Q} = \frac{XS}{(X + R)R}$$

$$\text{or, } \frac{P}{Q} = \frac{XS}{(X + S)\ R}$$

$$2 = \frac{3S}{3 + S}$$

$$\therefore \quad 6 + 2S = 3S$$

$$\therefore \quad S = 6\Omega$$

Hence, the value of resistance is 6Ω.

Q.41. In a potentiometer experiment, balance point with a cell of unknown e.m.f. was obtained at 55cm from one end and with a cell of e.m.f. 1.5V was got at 60cm from the same end. Calculate the unknown e.m.f.

Solution: Given,

First balanced length (l_1) = 55cm

Second balanced length (l_2) = 60cm

E_1 =?

E_2 = 1.5V

$$\text{Now, } \frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\text{or, } E_1 = E_2 \times \frac{l_1}{l_2} = 1.5 \times \frac{55}{60} = 1.375V$$

Hence, the unknown e.m.f. is 1.375V.

Q.42. A wire of length 10 m resistance 1.5Ω/m is connected to a cell of e.m.f. 4 V and internal resistance 1 Ω in series. What is the potential gradient of wire?

Solution: Given,

Length (l) = 10m

Resistance per unit length (R/l) = 1.5Ω/m

E.m.f. (E) = 4V

Internal resistance (r) = 1Ω

The resistance of the wire is 1.5 × 10 = 15Ω

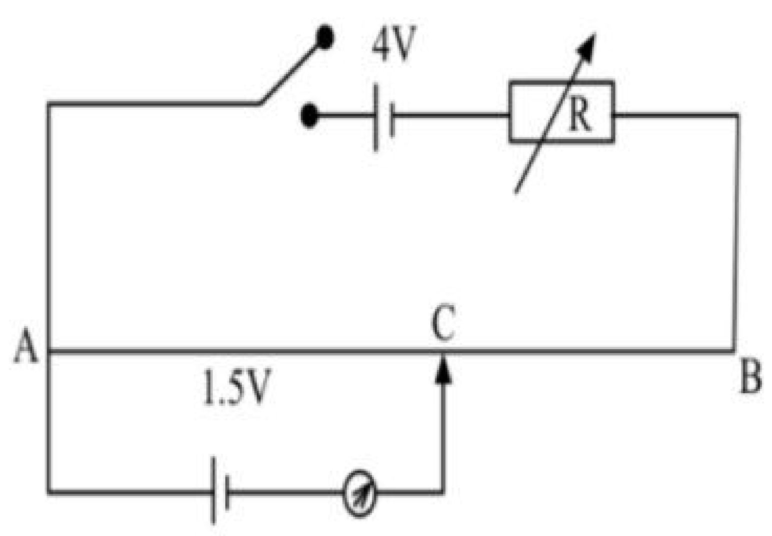
$$\text{Then, the current in the circuit (I)} = \frac{\text{emf}}{\text{Total resistance}} = \frac{4}{15 + 1} = 0.25A$$

$$\text{So the potential gradient is} = 0.25 \times \frac{15}{10} = 0.375V/m.$$

Hence, the potential gradient is 0.375V/m.

Q.43. A simple potentiometer circuit is set up as in the figure, using a uniform wire AB, 1.0m long which has a resistance of 2.0Ω. The resistance of the 4V battery is negligible. If the variable resistor R were given a value of 2.4Ω. What would be the length AC for zero galvanometer deflection?

If R were made 1.0Ω and the 1.5V cell and galvanometer were replaced by a voltmeter of resistance 20Ω, what would be the reading of the voltmeter if the contact C were placed at the mid-point of AB?



Solution:

For zero deflection in the galvanometer

$1.5 = I\ R_{AC}$

Where (I) is the current in the wire

$$1.5 = \frac{4}{R + R_{AB}} \times R_{AC}$$

$$\text{or,}\ 1.5 = \frac{4}{2.4 + 2} \times \frac{2.0}{1} \times l_{AC}$$

$$\text{or,}\ l_{AC} = \frac{1.5 \times 4.4}{4 \times 20} = 0.825\text{m}$$

The resistance of the wire AC when C at mid–point of AB (R) $= \frac{2}{2} = 1\Omega$.

∴ Combined resistance of the wire AC and the voltmeter resistance R_{AC} is given by

$$\frac{1}{R_{AC}} = \frac{1}{1} + \frac{1}{20} = \frac{21}{20}$$

$$\text{or,}\ R_{AC} = \frac{20}{21}\Omega$$

Current in the circuit,

$$I = \frac{4}{R + R_{AC} + R_{CB}}$$

$$I = \frac{4}{1 + \frac{20}{21} + 1} = \frac{4 \times 21}{62}$$

∴ Voltmeter reading,

$$V_{AC} = IR_{AC}$$

$$= \frac{4 \times 21}{62} \times \frac{20}{21} = 1.29\text{V}$$

Q.44. A galvanometer has an internal resistance of 1Ω. It gives maximum deflection for a current of 50mA. Show how this instrument can be converted into (i) a voltmeter with a maximum range of 2.5V (ii) an ammeter with a maximum range of 2.5A.

Solution: Given,

Resistance of galvanometer (G) = 1 Ω   Potential difference (V) = 2.5V

Current through galvanometer (I_g) = 50mA = 50×10^{-3}A = 0.05A

Current (I) = 2.5A

(i) To convert the galvanometer into voltmeter, a resistance R is connected in series. It is given by

$$R = \frac{V}{I_g} - G = \frac{2.5}{50 \times 10^{-3}} - 1 = 49\Omega$$

(ii) To convert the galvanometer into ammeter, a resistance S is connected in parallel and is given by

$$S = \frac{I_g \times G}{I - I_g} = \frac{0.05 \times 1}{2.5 - 0.05} = 0.02\Omega$$

Hence, the suitable resistance to convert galvanometer into voltmeter and ammeter are 49Ω and 0.02Ω.

Thermoelectricity

Q.45. **In a given thermocouple, the temperature of cold junction is -10°C, while the temperature of inversion is 510°C. What will be the neutral temperature?**

Solution: Given, temperature of cold junction (θ_c) =10°C

Temperature of inversion (θ_i) =510°C

Neutral temperature (θ_n) =?

We know, $\theta_n = \frac{\theta_c+\theta_i}{2} = \frac{10+510}{2} = 260°C$

Q.46. **The thermo e.m.f. and temperature of hot junction θ satisfy a relation E= aθ+bθ², Where a= 4.1×10⁵V (°C)⁻¹and b = -4.1×10⁻⁸V (°C)⁻². If the cold junction temperature is 0°C find the neutral temperature.**

Solution: Given equation is $E= a\theta+b\theta^2$ …(i)

$a= 4.1\times10^5 V (°C)^{-1}$, $b=-4.1\times10^{-8} V (°C)^{-2}$, cold junction temperature = 0°C

Now, $\frac{dE}{d\theta} = a + 2b\theta$

The neutral temperature is the temperature of hot junction at which thermo e.m.f. is maximum and $\frac{dE}{d\theta} = 0$

or, $a + 2b\theta = 0$

$a = -2b\theta$

$\theta_n = \frac{a}{-2b} = \frac{4.1\times10^{-5}}{-2\times(-4.1\times10^{-8})} = 500°C$

Q.47. **One junction of a thermocouple is at 0°C and the other at θ°C. The thermo e.m.f. (in volts) is given by $\varepsilon = 20 \times 10^{-6}\theta - 0.02 \times 10^{-6}\theta^2$. Find the neutral temperature and maximum value of e.m.f.**

Solution: Given,

$\varepsilon = 20 \times 10^{-6}\theta - 0.02 \times 10^{-6}\theta^2$

$\therefore \frac{d\varepsilon}{d\theta} = 20 \times 10^{-6} - 0.02 \times 10^{-6} \times 2\theta$

$= 2 \times 10^{-6} - 4 \times 10^{-8}\theta$

At neutral temperature θ_n,

Thermo e.m.f. is maximum

i.e. at $\theta = \theta_n$

Q.48. **If the temperature of the cold junction of a thermocouple is lowered, what will be the effect on its neutral temperature?**

Ans: The neutral temperature is the temperature of hot junction at which thermo e.m.f. becomes maximum. It depends only on the nature of metals forming thermocouple but it is independent of temperature of the cold junction. Therefore, lowering the temperature of cold junction does not bring any changes in neutral temperature.

Magnetic Field

Q.49. *What is the work done by the magnetic field on a moving charge?*

Ans: The Lorentz magnetic force experienced by the charge q moving with velocity v is given by

$F = Bqv \sin\theta$

Now, work done by the magnetic field on the charge is

$W = Fd\cos\theta$

For uniform magnetic field the charge particle transverses circular path.

i.e. $\cos\theta = \cos 90^0 = 0$

$W = 0$

Hence, no work is done by the magnetic field on the moving charge.

Q-50. ***An electron is moving northward with a velocity 10^7 ms^{-1} in a magnetic held of 3T directed eastwards. Calculate the instantaneous force on the electron. Given that charge on electron = 1.6×10^{-19} C.***

Solution: Here,

Electronic charge (e) = 1.6×10^{-19} C

Velocity of an electron (v) = 10^7 ms^{-1},

Magnetic field (B) = 3T

Angle between v and B is 90^o.

or, $F = qvB \sin 90^0$

$= 1.6 \times 10^7 \text{ ms}^{-1} \times 3 \times 1$

$= 4.8 \times 10^{-12}$ N

$\therefore$ Force is 4.8×10^{-12}N

Q-51. ***A 2MeV proton moves vertically downward though a uniform magnetic field of 2.5 Tesla.***

Calculate the force acting on the proton. (Given the mass of proton = 1.7×10^{-27} kg, charge on proton= 1.6×10^{-19} C)

Solution: Given,

Magnetic field (B) = 2.5 Tesla

Energy of proton (E) = 2 MeV = 2×10^6 eV = 3.2×10^{-13} J

Mass of proton (m) = 1.7×10^{-27} kg

Charge on proton (e) = 1.6×10^{-19} C

If v is velocity of proton, then $\frac{1}{2} m_p v^2 = KE$

or, $v = \sqrt{\frac{2E}{m_p}} = \sqrt{\frac{2 \times 3.2 \times 10^{-13}}{1.7 \times 10^{-27}}}$

$= 2 \times 10^7$ ms^{-1}

We have, the magnetic force on proton (F) = $Bev\sin\theta$

$\therefore F = 2.5 \times 1.6 \times 10^{-19} \times 2 \times 10^7 \sin 90^o$

$F = 8 \times 10^{-12}$ N

Hence, the force acting on proton is 8×10^{-12} N.

Q.52. *wire carrying current of 10 A and 2 m in length is placed in a field of flux density 0.34 T What is the force on the wire if is placed at 60^0 to the field?*

Solution: Given,

Length of the wire (l) = 2 m

Magnetic field applied (B) = 0.34T

Current flow through the wire (I) = 10 A

Angle between the direction of applied magnetic field and current (θ) = 60°.

We know that the force acting on the wire is given by

$$F = BI\,l\sin\theta = 0.34 \times 10 \times 2 \times \sin 60^\circ = 6.8 \times \frac{\sqrt{3}}{2} = 5.88\text{ N}$$

Q.53. *A copper wire has 1×10^{29} free electrons per cubic meter and cross sectional area $2mm^2$ carries a current of 6 A. Calculate the force acting on each electron if the wire is placed perpendicularly in a uniform magnetic field of flux density 0.1 T.*

Solution: Given,

No. of free electrons per m^3 (n) = $1 \times 10^{29}/m^3$

Cross section area (A) = $2\text{ mm}^2 = 2 \times 10^{-6}\text{ m}^2$

Current (I) = 6A

Flux density (B) = 0.1 T

Force on electron (F) =?

Now, The drift velocity $(v_d) = \frac{I}{enA}$

$$F = Bev_d = Be\frac{I}{enA} = \frac{BI}{nA} = \frac{0.1 \times 6}{1 \times 10^{29} \times 2 \times 10^{6}} = 3 \times 10^{-24}\text{ N}$$

Hence, force on each electron is 3×10^{-24}N

Q.54. *A vertical straight conductor X of length 0.5 m is situated in a uniform horizontal magnetic field of 0.T (i) Calculate the force on X when a current of 4 A is passed into it (ii) Through what angle must X be turned in a vertical plane so that the force on X is halved?*

Solution:

(i) Given

Current (I) = 4 A   Magnetic field (B) = 0.1 T

Length (l) = 0.5 m   Angle (θ)= 90°

Force (F) =?

We have, Force (F) = $BI\,l\sin\theta = 0.1 \times 4 \times 0.5 \times \sin 90^0 = 0.2$ N

(ii) Given

Current (I) = 4 A   Magnetic field (B) = 0.1 T

Length (l) = 0.5 m   Angle (θ') =?

$$F' = \frac{F}{2} = \frac{0.2}{2} = 0.1\text{ N}$$

Here, $F' = BI\,l\sin\theta'$

i.e. $$\theta' = \sin^{-1}\left[\frac{F'}{BIl}\right] = \sin^{-1}\left[\frac{0.1}{0.1 \times 4 \times 0.5}\right] = 60^\circ.$$

$\therefore\ \theta = 60^0$

Hence, the angle is 60°.

Q.55. *A straight horizontal rod X, of mass 50 gm and length 0.5 m is placed in a uniform magnetic field of 0.2 T perpendiculars to X. Calculate the current in X, if the force acting on it just balances its weight. ($g = 10\ N\ kg^{-1}$)*

Solution: Given,

Length (l) = 0.5m

Magnetic field (B) = 0.2 T

Weight (W) = $mg = 50 \times 10^{-3} \times 10 = 0.5$ N,

Force (F) = W = mg

Angle (θ) = 90°

Current (I) =?

We have,

$$\text{Force (F)} = BIl \sin\theta$$

$$mg = BIl \sin 90^\circ$$

$$\text{or,} \quad 0.5 = 0.2 \times I \times 0.5 \times 1$$

$$\therefore \quad I = \frac{1}{0.2} = 5\ A$$

$\therefore$ Current (I) = 5A.

Hence, the current in X is 5A.

Q.56. *A horizontal straight wire 5 cm long weight 1.2 g/m is placed perpendicular to a uniform horizontal magnetic field of flux density 0.6 T. If the resistance of the wire is 3.8Ω/m. Calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting.*

Solution: Given,

Length of the wire (l) = 5 cm = 0.05m

Magnetic field applied (B) = 0.6 T

Angle between the direction of magnetic field and current (θ) = 90^0

Resistance per unit length of the wire $\left(\frac{R}{l}\right)$ = 3.8Ω/m

Resistance of the wire (R) = $3.8 \times l = 3.8 \times 0.05 = 0.9\Omega$

Also mass per unit length of the wire $\left(\frac{m}{l}\right)$ = 1.2 g/m

Mass of the wire (m) = 1.2×0.05 g$= 6 \times 10^{-5}$ kg

We know that force acting on the wire is given by

$$F = BIl \sin\theta = B\left(\frac{V}{R}\right) l \sin\theta$$

$$\text{i.e.,} \quad V = \frac{FR}{B\,l \sin\theta} = \frac{mgR}{B\,l \sin\theta} \qquad \text{[Since: w = mg = F]}$$

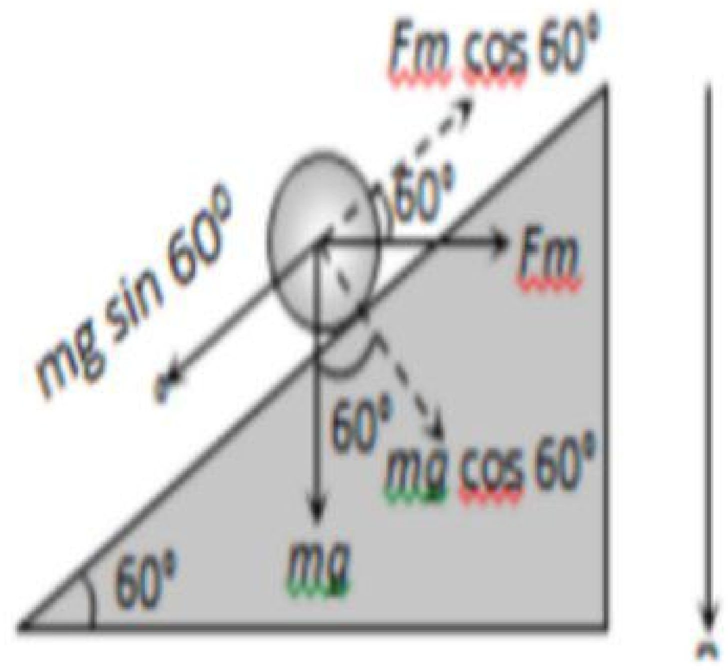
$$\therefore \quad V = \frac{mgR}{B\,l \sin\theta} = \frac{6 \times 10{-}5 \times 10 \times 0.19}{0.6 \times 0.05 \times \sin 90^0} = \frac{1.14 \times 10^{-4}}{0.03} = 3.8 \times 10^{-3} V$$

Hence, the patented difference is 3.8×10^{-3}V.

Hence, the current and voltage sensitivity changed are10:1 and 1:4 respectively.

Q.57. *A horizontal rod of mass 10 g and length 0.10 m is placed on a smooth plane inclined at 60^0 to the horizontal. A uniform vertical magnetic field of value B is applied in the region. Calculate B, if the rod remains stationary on the plane when a current of 1.73 A flows in the rod.*

Solution: Given,



Mass (m) = 10 gm　　Length (l)= 0.10 m

Angle (θ) = 60^0　　Current (I) = 1.73A

Magnetic field (B) =?

From figure,

$$BIl\cos60^o = mg\sin60^o$$

$$B \times 1.73 \times 0.1 \times \cos 60^o = 10 \times 10^{-3} \times 10 \times \sin 60^0$$

or, $$B = 10 \times 10^{-3} \times 10 \times \frac{\sqrt{3}}{2} \times \frac{1}{0.173} \times \frac{2}{1} = 1T.$$

Q.58. *A slice of indium antimonite is 2.5 mm thick and carries a current of 150 mA. A magnetic field of flux density 0.5T, correctly applied, produced a maximum Hall voltage of 8.75 mV between the edges of the slice. Calculate the number of free charge carriers per unit volume, assuming they each have a charge of -1.6×10^{-19}C.*

Solution: Given,

Thickness (t) = 2.5 mm = 2.5×10^{-3}m　　Current (I)= 150mA= 150×10^{-3}A

Magnetic field (B) = 0.5T　　Hall voltage (V_H) = 8.75mV=8.75×10^{-3}V

Electronic charge (e) = 1.6×10^{-19}C　　Electron density (n) =?

Now, we have,

$$V_H = \frac{BI}{net} \Rightarrow n = \frac{BI}{V_H et}$$

or, $$n = \frac{0.5 \times 150 \times 10^{-3}}{8.75 \times 10^{-3} \times (1.6 \times 10^{-19}) \times 2.5 \times 10^{-3}} = \frac{75}{35 \times 10^{-22}}$$

$\therefore$ $n = 2.14 \times 10^{22}$

Hence, the number of electrons are $2.14 \times 10^{22}/m^3$

Q.59. *A slab of copper, 2 mm thick and 1.50 cm wide, is placed in a uniform magnetic field of flux density 0.40 T, so that maximum flux passes through the slab. When a current of 75 A flows through it, a potential difference of 0.81 μV is developed between the edges of the slab. Find the concentration of the mobile electrons in copper.*

Solution: Given,

Thickness of slab (t) = 2 mm = 2×10^{-3}m   Width of slab (b) = 1.50 cm = 1.5×10^{-2}m

Magnetic flux density (B) = 0.40 T   Current in slab (I) = 75 A

Induced voltage (V_H) = 0.81 μV = 0.81×10^{-6}V   No of electron per m^3 (n) =?

We have,

$$V_H = \frac{IB}{net}$$

$$\text{or} \quad n = \frac{IB}{V_H et} = \frac{75 \times 0.40}{0.81 \times 10^{-6} \times 1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 1.15 \times 10^{29}/m^3.$$

Hence, the number of electrons are $1.15 \times 10^{29}/m^3$

Q.60. *A circular coil has 50 turns and a mean radius of 10 cm. If it carries a current of 3A, find the magnetic field at the centre of coil.*

Solution: Given,

Current (I) = 3 A   Radius (r) = 10 cm = 10×10^{-2} m = 0. 10m

Number of turns (N) = 50

Magnetic permeability of free space (μ_0) = $4\pi \times 10^{-7}$ Hm^{-1}

So the magnetic field at the centre of the coil is,

$$B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 50 \times 3}{2 \times 0.10} = 9.4 \times 10^{-4} \text{ Tesla}$$

Hence, the magnetic field at eh center of the coil is 9.4×10^{-4} Tesla.

Q.61. *In the Bohr model of the hydrogen atom the electron circulates around the nucleus in a path of radius 5.1×10^{-11} meter at a frequency of 6.8×10^{15} rev/sec. What value of B is set up at the centre of the orbit?*

Solution: Given

Radius (r) = 5.1×10^{-11} m   Frequency (f) = 6.8×10^{15} rev/s

For Hydrogen (q) = 1.6×10^{-19}

Then,

$$\text{Magnetic field (B)} = \frac{\mu_0 I}{2r} \qquad [I=q/t = e/t \text{ and } f=1/t]$$

$$= \frac{\mu_0 ef}{2r} = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 6.8 \times 10^{15}}{2 \times 5.1 \times 10^{-11}} = 14 \text{ T}$$

Hence, Magnetic field is 14T.

Q.61. *An alpha particle makes a full rotation in a circle of radius 1 meter in 2s. Calculate the value of magnetic field induction at the centre of the circle. ($\mu_o = 4\pi \times 10^{-7}$H/m)*

Solution: Given

Frequency (f) $= \frac{1}{T} = \frac{1}{2} = 0.5$ rev/s   Charge (q)= 2e

Then, Magnetic field (B) =?

We have,

$$B = \frac{\mu_0 I}{2r} \qquad [I=e/t \text{ and } f=1/t]$$

$$= \frac{\mu_0 ef}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 1.6 \times 10^{-19} \times 0.5}{2 \times 1} = 1.x\ 10^{-25}T$$

Hence, the magnetic field is 1.x 10^{-25}T.

Q.62. *A copper wire 28m long is wound into a flat circular coil 8.0cm in diameter. If the current of 4.50A flows through the coil, what is the magnetic induction at the centre?*

Solution: Given,

Length of the wire (l) = 28m

Radius of the coil (r) = Diameter/2 = 8/2 = 4cm= 0.04m

Current in the wire (I) = 4.5A

Then,

$2\pi r \times N = 28$

$N = 111$ turns

We know, $B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 111 \times 4.5}{2 \times 0.04} = 7.8 \times 10^{-3}T$

$\therefore \quad B = 7.8 \times 10^{-3}T$

Hence, the magnetic field is 7.8×10^{-3}T.

Q.63. *One-meter length of wire carrying constant current. The wire is bending to form circular loop. The magnetic field at the center of this circular loop is B. The same is now bending into circular loop of smaller size containing four turns in the loop. Calculate the magnetic field at the center of new loop.*

Solution: Given,

Length (l) = 1 m → radius of loop (r) $= \frac{l}{2\pi} = \frac{1}{2\pi}$

Magnetic field at center = B

We have, $B_{at\ center} = B = \frac{\mu_0 I}{2r} = \frac{\pi\mu_0 I}{l}$ ….(i)

When same wire is bend to form 4 loops coil.

$$B'_{at\ new\ center} = 4 \times \frac{\mu_0 I}{2r'} \qquad ….(ii)$$

Here $r' = \frac{l}{8\pi}$ then

$$B'_{at\ new\ center} = 4 \times \frac{\mu_0 I}{2\frac{l}{8\pi}} = 16 \times \frac{\pi\mu_0 I}{l} \qquad ….(iii)$$

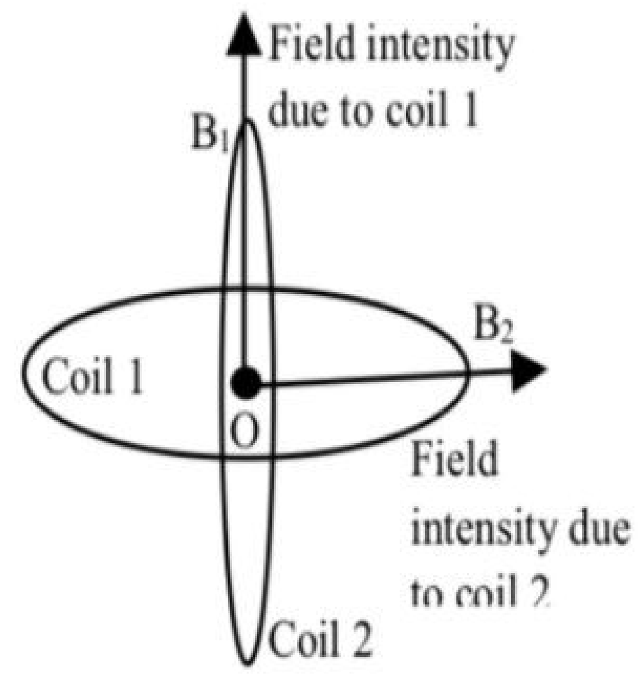
Dividing equation (iii) by (i)

$$\frac{B'_{at\ new\ center}}{B} = 16$$

$\therefore$ Magnetic field at new center (B') =16 B

Q.64. ***Two identical coils carrying equal currents have common center and their planes are right angled to each other. What is the ratio of magnitude of magnetic field and field due to one coil alone at the center?***

Solution: Two identical coils carrying equal currents have common center and their planes are right angled to each other,



The magnetic field at the center due to one coil (B_1) = $\frac{\mu_0 I}{2r}$ .... (i)

Since, they have common center, the magnetic field due to second coil at the center (B_2) = $\frac{\mu_0 I}{2r}$ .... (ii)

Because their planes are right angled to each other

Resultant magnetic field (B) = $\sqrt{B_1^2+B_2^2}$

$=\sqrt{2B_1^2}$

$=\sqrt{2}\,B_1$ .....(iii)

Then, the ratio of magnitude of magnetic field and field due to one coil alone at the center,

$$\frac{B}{B_1 \text{ or } B_2} = \frac{\sqrt{2}\,B_1}{B_1} = \sqrt{2}$$

∴ The ratio of magnitude of magnetic field and field due to one coil alone at the center is $\sqrt{2}$

Q.65. ***A horizontal wire of length 5 cm and carrying a current of 2 A is placed in the middle of a long solenoid at right angle its axis. The solenoid has 1000 turns per meter and carries a steady current I. Calculate I, if the force on the wire is vertically downwards and equal to 10^{-4} N.***

Solution: Given,

Length (l) = 5 cm = 5×10^{-2}m

Number of turns per meter (n) = 1000

Current (I_2) =?

Current (I_1) = 2 A

Force (F) = 10^{-4} N

If I be current flowing through solenoid, then the magnetic field at the wire due to current flow through the solenoid is given by,

$B = \mu_0 n I_1$

and $F = B I_2 l$

$F = (\mu_0 n I_1) I_2 l$

or, $I_2 = \frac{F}{\mu_0 n I I_1 l}$

$I_2 = \frac{10^{-4}}{4\pi \times 10^{-7} \times 1000 \times 2 \times 5 \times 10^{-2}} = 0.8$ A

∴ $I_2 = 0.8$A

Hence, the current is 0.8A.]

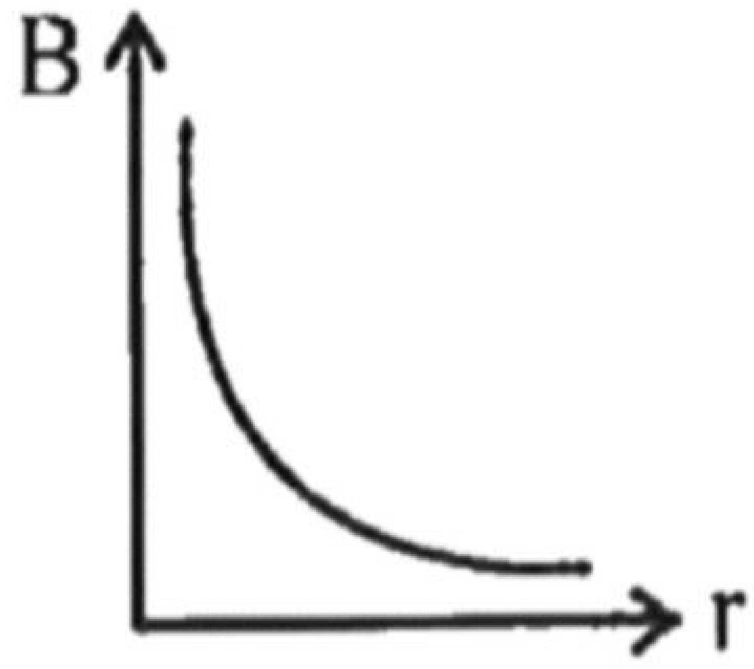


Figure 16.27: Magnetic field due to long straight current carrying conductor

Q.66. ***Calculate the magnetic field at the centre of coil in the form of a squire of side 4 cm carrying a current of 5 A.***

Solution

Magnetic field at 0 due to conductor PQ is

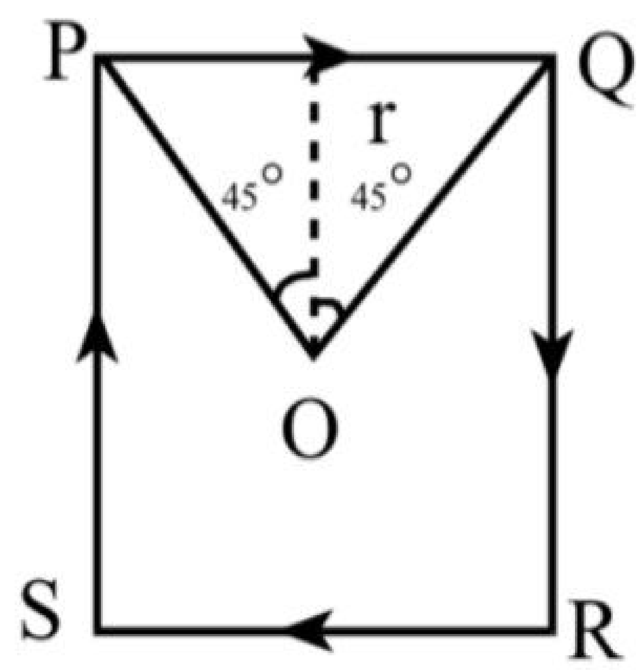
$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = 45°$

$I = 5$ A,

$r = 2$ cm $= 0.02$ m. So

$$B_1 = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{5}{0.02}[\sin 45° + \sin 45°] = \frac{5 \times 10^{-7}}{0.02} \times \frac{2}{\sqrt{2}} = 3.54 \times 10^{-5}\ T$$



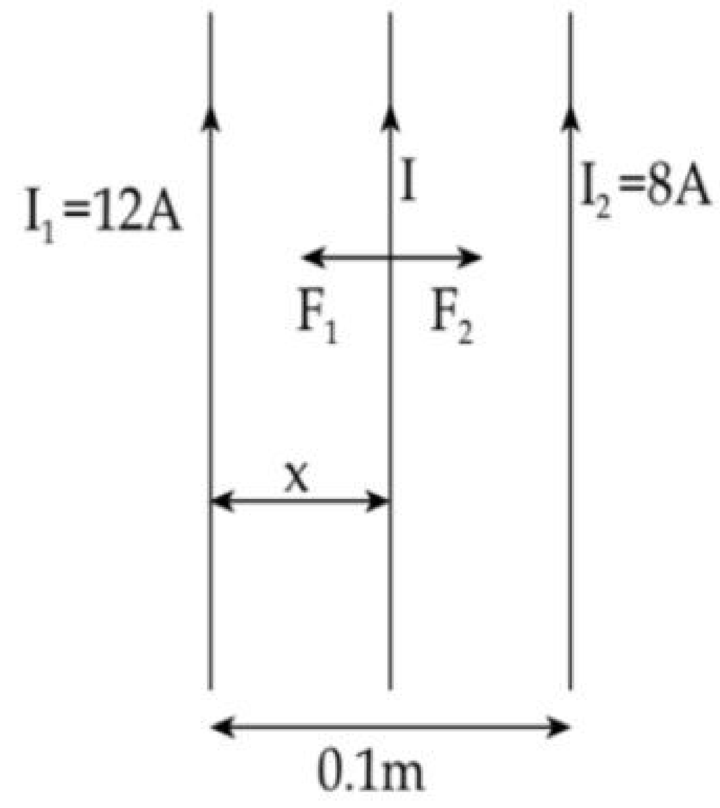
Net magnetic field intensity at 0 due to current carrying square

$B = 4\,B = 4 \times 354 \times 10^{-5}$ T

$\therefore\ B = 1.42 \times 10^{-4}$ T

Hence, the magnetic field is 1.42×10^{-4} T.

Q.67. ***Two long parallel conductors carry 12 A and 8 A current in the same direction. If the wire are10 cm apart, find where a third parallel wire also carrying a current must be placed so that the force experienced by it shall be zero.***



Solution:

Let, a third wire is placed between two wires at a distance x from the wire carrying 12 A so that force experienced by it per unit length will be zero.

i.e. $F_1 = F_2$

i.e. $\frac{\mu_0 I_1 I}{2\pi x} = \frac{\mu_0 I_2 I}{2\pi (0.1 - x)}$ where I is the current flow in third wire

But, $I_1 = 12$ A and $I_2 = 8$ A

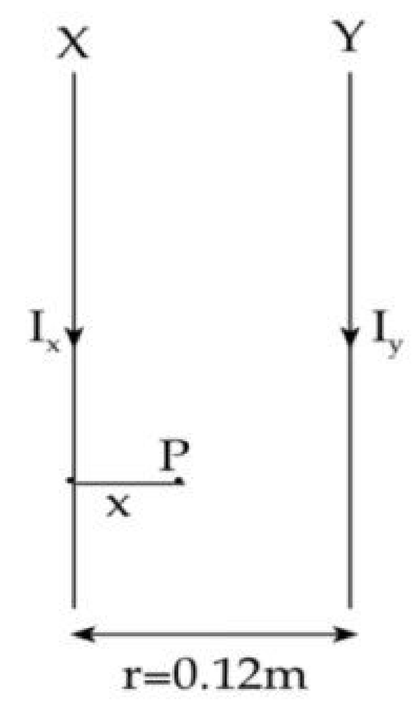
or, $\frac{12}{x} = \frac{8}{0.1 - x}$ or, $x = 0.06$ m

Hence, the third wire is should place at 6 cm from 1st wire.

Q.68. ***Two vertical conductor X and Y are 0.12 m apart and carry currents of 2A and 4A respectively in downward direction (a) ignoring earth's magnetic field, find***

(a) the distance from X of a point where the magnetic fields due to X and Y neutralize each other.

(b) calculate the force per meter on X and Y.



Solution, Given,

Current along X (I_x) = 2A   Current along Y (I_y) = 4A

Distance (r) = 0.12m

a) Let, 'P' be a point at a distance x from X where magnetic fields due to X and Y neutralize each other i.e. $B_x = B_y$

$$\frac{\mu_0 I_x}{2\pi x} = \frac{\mu_0 I_y}{2\pi (0.12 - x)}$$

or, $\frac{2}{x} = \frac{4}{0.12 - x}$

or, $x = 0.04$ m

b) Force per meter on X due to magnetic field of Y is given by,

$$\frac{F_X}{l_X} = B_Y I_X = \frac{\mu_0 I_Y}{2\pi r} I_X = \frac{4\pi \times 10^{-7} \times 4 \times 2}{2\pi \times 0.12} = 3.33 \times 10^{-6} N/m$$

Hence, the force per meter on X and Y is 3.33×10^{-6} N/m.

Magnetic Properties of Materials

Q.69. **Magnetic field B and magnetic intensity H in a material are found to be 2T and 950 Am^{-1} respectively. Calculate the μ_r and χ of the material. ($\mu_0 = 4\pi \times 10^{-7}$ Wb. A^{-1}M^{-1})**

Solution: Given,

Magnetic field (B) = 2 Tesla

Intensity (H) = 950 Am^{-1}

(μ_0) = $4\pi \times 10^{-7}$ Wb A^{-1}m^{-1}

We have, $B = \mu_0 (1 + \chi)H$

or, $(1 + \chi) = \dfrac{B}{H \times \mu_0}$

or, $\chi = \dfrac{B}{H \times \mu_0} - 1 = \left(\dfrac{2}{950 \times 4\pi \times 10^{-7}} - 1\right)$

$\therefore \chi = 1674$

Again we have, $\mu_r = (1 + \chi)$

or, $\mu_r = (1674 + 1)$

$\therefore \mu_r = 1675$

Electromagnetic Induction

Q.70. *Find the magnitude of induced e.m.f. in a 200 turn coil with a cross sectional area of 0.16 m^2, if the magnetic field through the coil changes from 0.10 Weber m^{-2} to 0.50 Weber m^{-2} at the uniform rate over a period 0.02 second.*

Solution: Given,

No. of turns (N) = 200 Area of coil (A) = 0.16m^2

Initial magnetic field (B_1) = 0.10 Weber m^{-2} Final magnetic field (B_2) = 0.50 Weber m^{-2}

Time (t) = 0.02 s

Initial value of magnetic flux linkage ($N\phi_1$) = NB_1A

Final value of magnetic flux linkage ($N\phi_2$) = NB_2A

Magnitude of the induced e.m.f

$$\varepsilon = \frac{N\phi_1 - N\phi_2}{t} = \frac{NB_2A - NB_1A}{t} = \frac{NA(B_2 - B_1)}{t}$$

$$= \frac{200 \times 0.16 \times (0.50 - 0.10)}{0.02} = \frac{200 \times 0.16 \times 0.40}{0.02} = 640 \text{ V}$$

Hence, induced e.m.f. is 640V.

Q.71. ***A magnetic flux passing perpendicular to the plane of a coil is given by $\phi = 4t^2+5t+2$, where ϕ is in Weber and t is in seconds. Calculate the magnitude of instantaneous e.m.f. induced in the coil when t =3sec.***

Solution: Given,

Flux in the coil (ϕ) = $4t^2+5t+2$

Time (t) =?

E.m.f. induced (ε) =?

We have, induced e.m.f (ε) = $\frac{d\phi}{dt} = \frac{d(4t^2 + 5t + 2)}{dt}$

$$\varepsilon = (8t + 5) = (8 \times 3 + 5) = 29V$$

Hence, the magnitude of induced e.m.f. is 29 V.

Q.72. ***A coil of 100 turns each area 2.0×10^{-3} m² has a resistance of 12Ω. It lies in a horizontal plane in a vertical magnetic flux density of 3.0×10^{-3} Wm⁻². What charge circulates through the coil if its ends are short circuit and the coil is rotated through 180° about a diametrical axis?***

Solution, Given,

Number of turns (N) = 100

Area of cross section (A) = $2.0 \times 10^{-3} m^2$

Magnetic induction (B) = 3.0×10^{-3} W/m²

Resistance (R) = 12Ω

When the coil is rotated through 180°, the direction of field is reserved,

$$\varepsilon = \frac{N\phi_1 - N\phi_2}{t} = \frac{NBA\cos 0 - NBA\cos 180}{t}$$

$$IR = \frac{NBA\,(1- (-1))}{t}$$

$$\frac{q}{t} R = \frac{2NBA}{t}$$

$$q = \frac{2NBA}{R}$$

[Simply, Charge circulated= $\frac{\text{Change in flux}}{\text{Resistance}}$]

$$\text{or, } q = \frac{2 \times 100 \times 2 \times 10^{-3} \times 3 \times 10^{-3}}{12}$$

or, $q = 10^{-4}$ C

∴ Charge (q) = 10^{-4} C

Hence, the charge circulated is 10^{-4} C.

Q.73. ***A horizontal straight wire 10 m long extending east and west is falling with a speed of 5.0 m s⁻¹ at right angles to the earth's magnetic field 0.30×10^{-4} Weber m⁻². What is the instantaneous value of the e.m.f. induced in the wire?***

Solution: Given,

Length of the wire (l) = 10 m

Velocity of the wire (v) = 5.0 m/s

Horizontal component of earth's magnetic field (B)= 0.30×10^{-4} Weber m⁻²

Now, $E = Bvl = 0.30 \times 10^{-4} \times 5.0 = 1.5 \times 10^{-3}$ V

Hence, the instantaneous value of the e.m.f. induced in the wire is 1.5×10^{-3} V

Q.74. ***Find in volts the e.m.f. induced in a straight conductor of length 20 cm on the armature of dynamo and 10 cm from the axis when the conductor is moving in a uniform radial field of 0.5 T and the armature is rotating at 1000 r.p.m.***

Solution: Given

Length (l) = 20 cm = 0.2 m

Radius (r) = 10 cm = 0.1 m

Magnetic inductile number of rev per min (B) = 0.5 T frequency (f) $= 1000$ r.p.m $= \frac{1000}{60}$ r.p.s

Induced e.m.f. (ε) =?

Since, $\varepsilon = Bvl$

or, $\varepsilon = Br\omega l$ $\quad (\therefore v = r\omega)$

or, $\varepsilon = Br2\pi f\, l$

or, $\varepsilon = 0.5 \times 0.1 \times 2\pi \times \frac{1000}{60} \times 0.2$

or, $\varepsilon = 1.05$ V

Hence the induced e.m.f is 1.05 V.

Q.75. ***A metal aircraft with a wing span of 40 m flies with a ground speed of 1080 km/hr in a direction eastward direction at constant altitude in a region of the northern hemisphere where the vertical component of the earth's magnetic field is 1.75×10^{-5} T. Find e.m.f. that developed between tips of the wing.***

Solution: Given,

Length (l) $= 40$ m

Velocity (v) $= 1080$ km/h $= \frac{1080 \times 1000}{60 \times 60} = 300$m/s

Vertical component of earth magnetic field (B_V) $= 1.75 \times 10^{-5}$ T

ε = ?

The induced e.m.f.

$\varepsilon = B_v v l = 1.75 \times 10^{-5} \times 40 \times 300$

$\therefore V = 0.21$V

Hence, the induced e.m.f. is 0.21V.

Q.76. ***A vertical metal disc, radius 8×10^{-2} m is rotating about its centre at 50 rev/s with its plane perpendicular to a horizontal magnetic field of 0.1 T. Find the e.m.f. between the centre and the rim.*** Solution: ***Given,***

Radius (r) $= 8 \times 10^{-2}$ m $\quad$ Frequency (f) = 50 rev/s

Flux density (B) = 0.1 T $\quad$ Induced e.m.f. (ε) =?

Since, $\varepsilon = Blv$

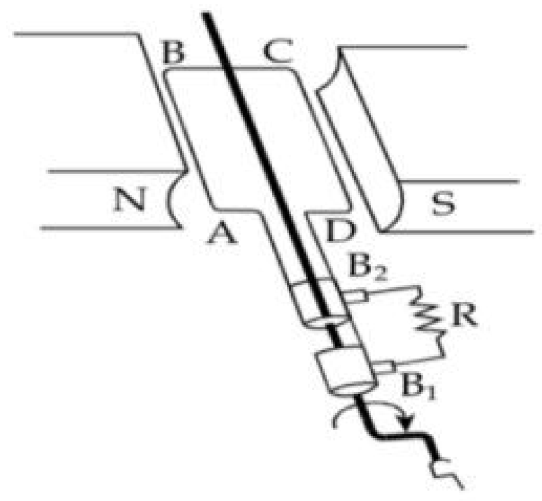
Here, $l = r$ and $v = \frac{0 + r\omega}{2} = \frac{r\omega}{2} = \frac{r\,2\pi f}{2}$

$$\therefore \quad \varepsilon = \frac{Br^2\, 2\pi f}{2} = \frac{0.1 \times (8 \times 10^{-2})^2\, 2\pi \times 50}{2} = 0.1 \text{ V}$$

Hence, the induced e.m.f. is 0.1 V.

Q.77. ***A rectangular coil of dimensions 0.10 m × 0.5 m consisting of 2000 turns rotates about an axis parallel to its long side, making 2100 revolutions per minute in a field of 0.1 T. What is the maximum e.m.f. induced in the coil? Also find the instantaneous e.m.f. when the coil is at 30^o to the field.***

Solution:



Number of turns of the coil (N) = 2000 turns

Area of the coil (A) = 0.10 m × 0.5 m = $0.05 m^2$

Frequency (f) = $\frac{2100}{60}$ per s.

Magnetic field (B) = 0.1T

Angular velocity (ω) = $2\pi f = \frac{2\pi \times 2100}{60}$ = 220 $rads^{-1}$

Maximum induced e.m.f., (ε_0)= NBAω = 2000 × 0.1× 0.05 × 220 =2200 V

Now, Induced e.m.f. when coil makes an angle of 30° with the field is given by

$\varepsilon = \varepsilon_0 \sin\omega t$

∴ ε = 2200 sin 30° = 1100 V

∴ Induced e.m.f. = 1100 V

Hence, the instantaneous induced e.m.f. is 1100 V.

Q.78 ***A coil of 50 turns has dimensions 10 × 10 cm. It is rotated at the rate of 5 rev/s in a uniform magnetic field of flux density 0.7 Tesla. What is the maximum e.m.f. induced in it?***

Solution: Given,

No. of turns (N) = 50

Rate of rotation (f) = 5 rev/s

Area of the coil (A) = 10 × 10 cm^2 = 100 × 10^{-4} m^2

Magnetic field of flux density = 0.7 Tesla

The induced e.m.f. in a rotating coil (ε) = NBA ω sin ωt

Here, angular velocity ω = $2\pi f = 2 \times \frac{22}{7} \times 5 = \frac{220}{7}$ s^{-1}

∴ ε_{max} = NBA ω = 50 × 0.7 × 100 × 10^{-4} × $\frac{220}{7}$ = 11 V.

Hence, the induced e.m.f. is 11 V.

Q.79. ***Current in a 10 milli Henry coil increases uniformly from zero to one ampere in 0.01 second. Find the value of self-induced e.m.f.***

Solution:

Self-inductance of the coil,

Self-inductance (L) = 10 milli Henry= 10^{-2} Henry

Initial current in the coil = 0

Final current in the coil = 1 ampere

∴ Change in current. dI = 1 – 0 = 1 ampere

Time in which the current changes, dt = 0.01 s

We know that magnitude of induced e.m.f., ε $= -L\frac{dI}{dt}$ $= 10^{-2} \times \frac{-1}{0.01}$ = –1 V

The self-induced e.m.f. will act so as to oppose the growth of current.

Q.80. ***Two closely circuit coil has the same number of turns but one has twice the radius of the other. What is the ratio of self inductance of two coils?***

Ans: The magnetic produced in the coil is given by

ϕ = BA for circular coil $B = \frac{\mu_0 I}{2r}$

$\therefore \quad \phi = \frac{\mu_0 I.A}{2r} = \frac{\mu_0 I}{2r} \times \pi r^2$

$LI = \frac{\mu_0 \pi r}{2}$

for 1st coil, $L_1 I = \frac{\mu_0 \pi r_1}{2}$ ....(i)

for 2nd coil, $L_2 I = \frac{\mu_0 \pi r_2}{2}$ ....(ii)

Dividing equation (i) by (ii)

$\frac{L_1}{L_2} = \frac{r_1}{r_2} = \frac{1}{2}$

$\therefore \quad L_1 : L_2 = 1 : 2$

Q.81. ***A long solenoid is wound with 500 turns per meter and the current in its windings is increasing at the rate of 1sw 00A/s. The cross-sectional area of the solenoid is 4.0 cm². (a) Find the magnitude of the induced e.m.f. in the wire loop outside the solenoid. (b) Find the magnitude of the induced electric filed within the loop if its radius is 2.0 cm.***

Solution: Given,

Number of turns per meter (n) = 500
Cross-sectional area (A) = 4 cm² = 4×10^{-4} m²
The rate of increasing of current $\left(\frac{dI}{dt}\right)$ = 100A/s
Radius (r) = 2 cm = 2×10^{-2} m

Then we have,

$\varepsilon = \frac{-d\phi}{dt}$

(a) Again we have,

$\phi = BA$

$\therefore \quad \varepsilon = \frac{-d(BA)}{dt}$

For long solenoid, magnetic field (B) $= \mu_0 nI$

$\therefore \quad \varepsilon = -\mu_0 nA\frac{dI}{dt}$

or, $\varepsilon = -4\pi \times 10^{-7} \times 500 \times 4 \times 10^{-4} \times 100$

$\therefore \quad \varepsilon = -25 \times 10^{-6} V = -25 \mu V$.

(b) $E = \frac{\varepsilon}{l}$ for solenoid, $l = 2\pi r$

$\therefore \quad E = \frac{25 \times 10^{-6}}{2\pi\, (2.0 \times 10^{-2})} = 2 \times 10^{-4} V/m$

Q.82. A current of 10 A in the primary of a circuit is reduced to zero at a uniform rate in 10^{-3} second. If the co efficient of mutual inductance is 3 Henry, what is the induced e.m.f. in the secondary?

Solution: Given,

	Initial current in the primary coil (I_1)	= 10 A
	Final current in the primary coil (I_2)	= 0
∴	Change in current in the primary coil (dI)	= 10 – 0 = 10 A
	Time in which the current changes (dt)	= 10^{-3} s
	Mutual inductance (M)	= 3 H

The magnitude of induced e.m.f. is given by

$$\varepsilon = M\frac{dI}{dt} = 3 \times \frac{10}{10^{-3}} = 30000 \text{ V}$$

∴ Induced e.m.f. (ε) = 30000 V

Q.83. The e.m.f. induced in the secondary is 288 V while the primary current changes from 0 to 9.0 A in 0.025 s. Find the mutual inductance between the coils.

Solution: Given,

Change in current (dI_p) = 9 – 0 = 9 A

Change in time (dt)= 0.025 sec

E.m.f. induced (ε) = 288 V

Mutual inductance (M) =?

We have,

$$\varepsilon_s = M\frac{dI_P}{dt}$$

or, $$M = \frac{288 \times 0.025}{9} = 0.8 \text{ H}$$

Hence, the mutual inductance is 0.8 H.

Q.84. A plane circular coil has 200 turns and its radius is 0.10m. It is connected to a battery after switching on the circuit a current of 2A is set up in the coil. Calculate the energy stored in the coil.

Solution: Given

No. of turns of the coil (n) = 200 turns

Radius of the coil (r)= 0.10m

Set up current (I) = 2A

Energy stored in the coil (U) =?

Now,

$$U = \frac{1}{2}LI^2 \qquad [\phi = BA, B = \frac{\mu_0 NI}{2r}, A = \pi r^2]$$

$$= \frac{1}{2}\frac{\mu_0 N^2}{2r}I^2A = \frac{1}{2}\frac{\mu_0 N^2}{2r}I^2\,\pi r^2 = \frac{\mu_0 N^2}{4}I^2\,\pi r = \frac{1}{4}\times 4\pi\times 10^{-7}\times(200)^2\times(2)^2\times\pi\, 0.1$$

Hence, the energy stored in the coil is 1.58×10^{-2}J.

Q.85. *In a step up transformer, the transformation ratio is 100. The primary voltage is 220 volt and input is 1100 W. The number of turns in primary is 100. Calculate (i) the number of turns in the secondary (ii) the current in the primary (iii) the voltage across the secondary and (iv) the current in the secondary.*

Solution: Given,

Transformation ratio (k) = 100
Number of turns in primary coil (N_p) = 100

Now, $\frac{N_s}{N_p} = k$

$\therefore \quad N_S = k \times N_p = 100 \times 100 = 10000$

(ii) Input power, $E_p I_p = 11000$ watt, $E_p = 220$ V

$$I_p = \frac{1100}{E} = \frac{1100}{220} = 5 \text{ A}$$

(iii) Also $\frac{E_s}{E_p} = k$

$E_s = k.\ E_p = 100 \times 220 = 22000$ V

(iv) Also, $\frac{E_s}{E_p} = \frac{I_p}{I_s}$

$$I_s = \frac{E_p}{E_s} \times I_p = \frac{220}{22000} \times 5 = 0.05\text{A}$$

Hence, the current in the secondary is 0.05A.

Alternating Current

Q.86. *The equation of alternating current of a circuit is given by $I = 50\sin 100\pi t$. Find (i) frequency of AC applied (ii) mean value of current during positive half cycle (iii) virtual value of current and (iv) the value of current 1/300 second after it was zero.*

Solution: The value of current in an AC circuit is given by $I = I_0 \sin\omega t = I_0 \sin 2\pi f t$

Given,

Current (I) = $50 \sin 100\pi t$

Comparing the two equations, we have

$2\pi f t = 100\pi t$................(i)

And $I_0 = 50$ A

Using these equations, we have

(i) Frequency, $(f) = \frac{100\pi t}{2\pi t} = 50$ Hz

(ii) Mean value of AC, $(I_m) = 0.636\, I_o$

$= 0.636 \times 50 = 31.8$ A

(iii) Virtual value AC $(I_v) = 0.707\, I_o = 0.707 \times 50 = 35.35$ A

(iv) The value of current at t (I) = 1/300 s is given by

$= 50 \sin 100 \times \pi \times 1/300$

$= 50 \sin\frac{\pi}{3} = 43.3$ A

Hence, the current is 43.3 A.

Q.87. *Alternating voltage in an AC circuit is represented by $V = 100\sqrt{2}\sin(100\pi t)$ volts. Find its root mean square value and the frequency.*

Solution: Given,

Potential is given by

$V = 100\sqrt{2}\sin(100\pi t)$ .... (i)

Root mean square value of potential =?

Frequency (f) =?

Comparing with standard equation

$V = V_0 \sin\omega t$ .... (ii)

We get, peak value of potential

$V_0 = 100\sqrt{2}$

Rms potential $= \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100V$

For frequency,

$\omega = 100\pi$

or, $2\pi f = 100\pi$

$\therefore$ $f = 50Hz$

Hence, the rms value of potential and frequency are 100V and 50 Hz respectively.

Q.88. *An inductance of negligible resistance, whose reactance is 22 Ω at 200 Hz, is connected to a 220 volt, 50 hertz power line. What is the value of the inductance and current?*

Solution:

The inductive reactance at 200 Hz (X_L) = 22 ohm

Thus, $X_L = 2\pi f L = 22$

$$L = \frac{22}{2\pi f} = \frac{22}{2 \times \frac{22}{7} \times 200} = \frac{7}{400} = 0.0175 \text{ H}$$

The inductive reactance of the coil at 50 Hz

$$X_L = 2\pi f L = \frac{2\pi \times 50 \times 7}{400} = \frac{2 \times 22 \times 50 \times 7}{7 \times 400} = 5.5\ \Omega$$

And current,

$$I = \frac{V}{X_L} = \frac{220}{5.5} = 40A$$

Hence, the current is 40A.

Q.89. *When 100-volt DC is applied across a coil, a current of 1 ampere flows through it. When 100-volt AC at 50 cycles s^{-1} are applied to the same coil only 0.5 ampere current flows. Calculate the resistance, the impedance and the inductance of the coil.*

Solution:

In DC circuit,

Resistance, $R = \frac{V}{I} = \frac{100}{1} = 100\,\Omega$

In AC circuit impedance

$Z = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$

For a LR circuit

$$\therefore \quad Z = \sqrt{R^2 + X_L^2}$$

$$X_L^2 = Z^2 - R^2 = (200)^2 - (100)^2$$

$$= 40000 - 10000 = 30000$$

$$\therefore \quad X_L = \sqrt{3} \times 100 = 173.2\,\Omega$$

But, $X_L = 2\pi f L$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{173.2}{2 \times 3.14 \times 50} = 0.55 \text{ H}$$

Hence, the inductance is 0.55 H

Q.90. *An alternating voltage of 10 V rms and frequency 50 Hz is applied to (i) a resistor of 5 Ω (ii) an inductor of 2H and (iii) a capacitor of 1µF. Determine the rms current following in each case.*

Solution

(i) Rms value of e.m.f. (V_{rms}) = 10 V

Resistance (R) = 5Ω

Rms value of current in resistor (I_{rms}) = $\frac{V_{rms}}{R} = \frac{10}{5} = 2$ A

(ii) Inductance (L) = 2H

Rms value of current in inductor (I_{rms}) = $\frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L} = \frac{10}{2\pi \times 50 \times 2} = 0.016$ A

(iii) Capacitance (C) = 1µF = 10^{-6} F

Rms value of current in capacitor $I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{2\pi f C} = 10 \times 2\pi \times 50 \times 10^{-6} = 0.0031$ A

Hence, the rms value of current is 0.0031 A.

Q.91. *AC mains of 200 V, 50 Hz is joining to a circuit containing an inductance of 100 mH and a resistance of 20 Ω in series. Calculate the power consumed.*

Solution: Given,

E.m.f. (V) = 200 V

Inductance (L) = 100 mH = 0.1 H

Power consumes (P) =?

Frequency (f) = 50 cycles/sec

Resistance (R) = 20 Ω

The impedance of L - R circuit is

$$Z = \sqrt{R^2 + (L\omega)^2}$$

$$\text{or,} \quad Z = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(20)^2 + (2\pi \times 50 \times 0.1)^2}$$

$$\text{or,} \quad Z = 37.24\Omega$$

The circuit is given by,

$$I = \frac{V}{Z} = \frac{200}{37.24} = 5.37\text{A}$$

Power consumed, $P = I^2R = (5.37)^2 \times 20 = 576.74$ W

Hence, the power consumed is 576.74 W.

Q.92. *A 50 V, 50 Hz AC supply is connected to a resistor of resistance 40Ω in series with a solenoid whose inductance is 0.2 H. The potential difference between the ends of the resistor is found to be 20 V. What is the resistance of the wire of the solenoid? (Assume $\pi^2 = 10$)*

Solution: Given,

E.m.f. (V) = 50 V   Frequency (f) = 50 Hz
Inductance (L) = 0.2 H   Resistance (R) = 40Ω
Potential across resistance (V_R) = 20 V   Internal resistance of coil (r) = ?

Let 'r' be the resistance of the solenoid then the impedance of the circuit is,

$$Z = \sqrt{(R + r)^2 + X_L^2}$$

$$\text{or, } Z = \sqrt{(40 + r)^2 + (2\pi fL)^2} = \sqrt{(40 + r)^2 + 4 \times 10 \times 2500 \times 0.04} = \sqrt{(40 + r)^2 + 4000}$$

The current through the circuit is,

$$I = \frac{V_R}{R} = \frac{20}{40} = 0.5 \text{ A}$$

Impedance of circuit is

$$Z = \frac{V}{I} = \frac{50}{0.5} = 100\,\Omega$$

i.e. $100 = \sqrt{(40 + r)^2 + 4000}$
or, $(40 + r)^2 + 4000 = 10000$
or, $(40 + r)^2 = 6000$
or, $40 + r = 77.5$
or, $r = 37.5\,\Omega$

Hence, the internal resistance of coil is 37.5 Ω.

Q.93. ***A coil having inductance and resistance is connected to an oscillator giving a fixed sinusoidal output voltage of 5 V rms with the oscillator set at a frequency of 50Hz the rms current in the coil is 1 A and at a frequency of 100 Hz, the rms current 0.625 A. (a) Determine the inductance of the coil, (b) calculate the ratio of the powers dissipated in the coil in the two cases.***

Solution: Given,

E.m.f. (V) = 5V   Frequency (f_1) = 50 Hz
Current (I_1) = 1 A   Frequency (f_2) = 100 Hz
Current (I_2) = 0.625 A

(a) Inductance (L) =?

The impedance of circuit at frequency 50 Hz is

$$Z_1 = \frac{V}{I_1} = \frac{5}{1} = 5\,\Omega$$

The impedance of circuit of frequency 100 Hz is

$$Z_2 = \frac{V}{I_2} = \frac{5}{0.625} = 8\,\Omega$$

Since, $Z_1 = \sqrt{R^2 + \omega_1 L^2}$
or, $Z_1^2 = R^2 + \omega_1^2 L^2$
or, $Z_1^2 = R^2 + 4\pi^2 f_1^2 L^2$ …………… (i)
Similarly, $Z_2^2 = R^2 + 4\pi^2 f_2^2 L^2$ ……....… (ii)
Subtracting equation (ii) from (i) we get

$$Z_2^2 - Z_1^2 = (4\pi^2 f_2^2 - 4\pi^2 f_1^2)L$$

$$\text{or, } L = \frac{Z_2^2 - Z_1^2}{4\pi^2 (f_2^2 - f_1^2)} = \frac{(8)^2 - (5)^2}{4\pi^2[(100)^2 - (50)^2]} = 1.32 \times 10^{-4}\text{H}$$

b) Since, $P_1 = I_1^2 R$ and $P_2 = I_2^2 R$

$$\frac{P_1}{P_2} = \frac{I_1^2}{I_2^2} = \left(\frac{1}{0.625}\right)^2 = 2.56$$

Hence, the ratio of power is 2.56.

Q.94. *An inductor of 80 mH is in series with a 20 Ω resistor and a 100 V (rms) 50 Hz AC source. Calculate (a) the rms current (b) the phase angle between the current and applied voltage (c) the power factor.*

Solution: Given

Alternating e.m.f. (V) = 100 V Frequency (f) = 50 Hz

Inductance (L) = 80 mH = 80×10^{-3} H Resistance (R) = 20 Ω

a) Alternating current (I_{rms}) =?

$$I_{rms} = \frac{V}{Z} = \frac{100}{\sqrt{R^2 + (\omega L)^2}} = \frac{100}{\sqrt{(20)^2 + (2\pi f L)^2}}$$

or, $$I_{rms} = \frac{100}{\sqrt{(20)^2 + (2\pi \times 50 \times 820 \times 10^{-3})^2}}$$

or, I_{rms} = 3.1 A

b) Power factor (cos ϕ) =?

$$\tan\phi = \frac{IX_L}{IR}$$

or, $$\tan\phi = \frac{X_L}{R} = \frac{L\omega}{R} = \frac{2\pi f L}{R} = \frac{5.1}{20}$$

or, $\tan\phi = 1.255$

or, $\phi = 51.5^o$

Power factor $\cos\phi = \cos 51.5^o = 0.6225$

Hence, the power factor is 0.6225.

Q.95. *If the source is 10 V, frequency 1 kHz, and the capacitance 2 μF, what value of R in the circuit would allow a current of 0.1 A to flow?*

Solution: Given,

E.m.f. (V) = 10 V Frequency (f) = 1 kHz = 10^3 Hz

Capacitance (C) = 2 μF = 2×10^{-6} F Current (I_v) = 0.1 A

If Z is impedance of CR circuit, then

Impedance (Z) $= \frac{E_V}{I_V} = \frac{10}{0.1} = 100\ \Omega$

Now, $$Z = \sqrt{R^2 + \frac{1}{C^2\omega^2}} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

Resistance (R) $$= \sqrt{Z^2 - \frac{1}{4\pi^2 f^2 C^2}}$$

$$= \sqrt{100^2 - \frac{1}{4 \times 9.87 \times (10^3)^2 \times (2 \times 10^{-6})^2}}$$

$$= \sqrt{100^2 - 6332.3} = 60.56\ \Omega$$

Hence, the resistance is 60.56Ω.

Q.96. *Calculate the reactance of an inductor L of inductance 100 mH and of a capacitor C of capacitance 2μF both at frequency of 50 Hz. At what frequency their reactances are equal in magnitude?*

Solution: Given,

Inductance (L) $= 100\text{ mH} = 100 \times 10^{-3} = 0.1\text{H}$

Capacitance (C) $= 2\mu\text{F} = 2 \times 10^{-6}\text{ F}$

Frequency (f) $= 50\text{ Hz}$

Resonating frequency (f_0) $=?$

Reactance of inductor (X_L) $= \omega L = 2\pi f L = 2\pi \times 10 \times 0.1 = 31.4\Omega$

Reactance of capacitor (X_C) $= \dfrac{1}{\omega C} = \dfrac{1}{2\pi f L} = \dfrac{1}{2\pi \times 50 \times 2 \times 10^{-6}} = 1591.5\,\Omega$

When, $f = f_0$, $X_L = X_C$

or, $2\pi f_0 L = \dfrac{1}{2\pi f_0 c}$

or, $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 10^{-6}}} = 355.9\text{ Hz}$

Hence, the frequency is 355.9 Hz.

Q.97. ***A constant AC supply is connected to a series circuit consisting of a resistance of 300Ω in series with a capacitance 6.67μF, the frequency of the supply being 3000/2π Hz. It is desired to reduce the current in the circuit to half its value. Show how this could be done by placing an additional resistance.***

Solution: Given,

Resistance (R) $= 300\Omega$ | Capacitance (C) $= 6.67\mu\text{F} = 6.67 \times 10^{-6}\text{F}$

Frequency (f) $= \dfrac{3000}{2\pi}\text{Hz}$ | Initial current (I) = I (Let)

Final current (I') $= \dfrac{I}{2}$

First

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} \quad \text{............. (i)}$$

$$I' = \frac{V}{\sqrt{(R+r)^2 + X_C^2}}$$

$$\frac{I}{I'} = \frac{\sqrt{R^2 + X_c^2}}{\sqrt{(R+r)^2 + X_C^2}}$$

$$4(R^2 + X^2_C) = (R + r)^2 + X^2_C \qquad [I' = \frac{I}{2}]$$

$$4R^2 + 4X^2_C = (R + r)^2 + X^2_C$$

$$4R^2 + 3X^2_C = (R + r)^2$$

$$(R + r)^2 = \left[4 \times (300)^2 + 3 \times \frac{1}{4\pi^2\left(\frac{3000}{2\pi}\right)^2 \times (6.67 \times 10^{-6})^2}\right] = 4 \times (300)^2 + \frac{3 \times 10^{12}}{4 \times 10^8} = [4 \times (300)^2 + (7500)]$$

$$(R + r)^2 = 6062\Omega$$

$$\therefore \quad r = 6062.2 - 300 = 306.2\Omega$$

Hence, the additional resistance is 306.2Ω.

Q.98. ***An iron cored coil of 2 H and 50 Ω resistance placed in series with a resistor or 450 Ω and 200 V, 50Hz AC supply is connected across the arrangement, find***

(i) the current flowing the coil

(ii) it's phase angle relative to the voltage supply

(iii) the voltage across the coil

Solution: Given,

Inductance (L) = 2H Internal resistance (r) = 50Ω

Resistance (R) = 450 Ω Voltage (V) = 200V

Frequency (f) = 50Hz Current (I) =?

Phase (ϕ) = ? Voltage across the coil (V_L) =?

We have,

$$\text{Current (I)} = \frac{V}{Z}$$

$$I = \frac{V}{\sqrt{(R+r)^2+X_L{}^2}} = \frac{V}{\sqrt{(R+r)^2+4\pi^2f^2L^2}}$$

$$\text{or,} \quad I = \frac{200}{[(450+50)^2 + 4\pi^2(50)^2 \times 2^2]^{1/2}}$$

$$= \frac{200}{[644784]^{1/2}} = \frac{200}{803}$$

∴ Current (I) = 0.25A

$$\text{Again, } \tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi fL}{R}$$

$$\phi = \tan^{-1}\left(\frac{2\pi \times 50 \times 2}{450}\right) = \tan^{-1}(1.39) = 51.8°$$

∴ ϕ = 51.8°

Voltage across the coil (V_L) = IX_L = 0.25 × 2π × 50 × 2 = 157V

Hence, the current, phase angle and voltage are 0.25A, 51.8° and 157V respectively.

Q.99. ***L-C-R alternating current series circuit of L = 1H, C = 1μF and R = 100Ω are connected in series with a source of frequency 50Hz. What is the phases shift between current and voltage?***

Solution: Given,

Inductance (L) = 1H Capacitance (C) = 1μF = 10^{-6}F

Resistance (R) = 100Ω Frequency (f) = 50Hz

Phase (ϕ) =?

We know,

$$\tan\phi = \pm\left(\frac{X_L - X_C}{R}\right)$$

$$= \pm\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \tan\phi = \pm\frac{\left[2\pi \times 50 \times 1 - \frac{1}{2\pi \times 50 \times 10^{-6}}\right]}{100} = \pm\frac{\left[100\pi - \frac{10^6}{100\pi}\right]}{100}$$

$$\text{or,} \quad \tan\phi = \frac{2868.94}{100}$$

or, tanϕ = 29.69

Phase (ϕ) = $\tan^{-1}$(29.69)

∴ ϕ = 88°

Hence, phase shift between current and potential is 88°

Q.100. ***An iron cored coil of inductance 3H and 50Ω resistance is placed in series with a resistor of 550Ω, and a 100V, 50Hz ac supply is connected across the arrangements. Find the current following in the coil and the voltage across the coil.***

Solution: Given,

Inductance (L) = 3H   Internal resistance (r) = 50Ω

Resistance (R) = 550Ω   Voltage (V) = 100V

Frequency (f) =50Hz   Current (I) =?

Voltage across the coil (V_L) =?

We have,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R+r)^2 + X_L^2}}$$

$$I = \frac{100}{\sqrt{(R + r)^2\ 4\pi f^2 L^2}} = \frac{100}{\sqrt{(550 + 50)^2 + 4\pi^2(50)^2 \times 3^2}} = \frac{100}{\sqrt{1248264.39}} = \frac{100}{1117.25}$$

$\therefore$ I = 0.09A

Voltage across the coil (V_L) = IX_L = $I \times 2\pi f L$ = $0.09 \times 2\pi \times 50 \times 3$ = 85V

$\therefore$ V_L = 85V

Hence, the current and voltage across the coil are 0.09A and 85V respectively.

Q.101. ***An inductor, a resistor and a capacitor are connected in series across an AC circuit. A voltmeter reads 60V when connected across the inductor, 16V across the resistor and 30V across the capacitor:***
(i) What will the voltmeter read when placed across the series circuit?
(ii) What is the power factor of the circuit?

Solution: Given,

Potential across the inductor (V_L) = 60V   Potential across the resistor (V_R) = 16V

Potential across the capacitor (V_C) = 30V   Power factor (cosϕ) = ?

E.m.f. of circuit

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = [(16)^2 + (60 - 30)^2]^{1/2}$$

$\therefore$ V = 34V

From phase diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{60 - 30}{16} = 1.875$$

$\phi = \tan^{-1}(1.875)$

$\therefore$ $\phi = 62°$

The power factor (cosϕ) = cos62° = 0.47

Hence, the power factor is 0.47.

Q.102. ***A circuit consists of an inductor of 200μH and resistance of 10Ω in series with a variable capacitor and a 0.10V (rms), 1.0 MHz supply. Calculate (i) the capacitance to give resonance (ii) quality factor of the circuit at resonance.***

Solution: Given,

Inductance (L) $= 200\mu H = 200 \times 10^{-6} H = 2 \times 10^{-4} H$

Resistance (R) $= 10\ \Omega$

Capacitance (C) =?

Potential across capacitor (V_C) = 0.1V

Quality factor (Q) =?

Frequency (f) $= 1\ MHz = 10^6 Hz$

At resonance $X_L = X_C$

$$\therefore \quad \omega L = \frac{1}{\omega C}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{or,} \quad 10^6 = \frac{1}{2\pi(2 \times 10^{-4})^{1/2}C^{1/2}}$$

$$\text{or,} \quad C^{1/2} = \frac{1}{2\pi \times \sqrt{2 \times 10^{-4}} \times 10^6} = 1.125 \times 10^{-5}$$

$$\therefore \quad C = 1.27 \times 10^{-10}\ F$$

Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2 \times 10^{-4}}{1.26 \times 10^{-10}}}$$

$$\therefore \quad Q = 126$$

Hence, the capacitance and quality factor are 1.27×10^{-10}C and 126 respectively.

Electrons

Q.103. ***An oil drop of radius 10^{-6} m carrying a certain charge remains stationary between the two horizontal plates. Find the number of excess electrons in the oil drop, if the applied electric field is 1.7×10^5 V/m. The density of oil is 2.0×10^3 kgm^{-3}.***

Given; radius (r) = 10^{-6} m

electric field strength (E) = 1.7×10^5 V/m

density of oil (ρ) = 2.0×10^3 kg m^{-3}

number of electrons (n) = ?

When oil drop remains stationary between two horizontal plates having electric field

Weight = Electrostatic force

$$mg = qE$$

$$\frac{4}{3}\pi r^3\rho g = ne\,E$$

$$n = \frac{4}{3eE}\pi r^3\rho g = \frac{4\times3.14\times(10^{-6})^3\times2\times10^3\times9.8}{3\times1.6\times10^{-19}\ \times1.7\times10^5} = 3.0$$

There are 3 electrons in the oil droplet.

Q.104. ***Two plane metal plates 5.0 cm long are held horizontally 3.0 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 400 V and the lower is earthed. Electrons having a velocity of $1.0 \times 10^7 m/s$ are injected horizontally midway between the plates and in a direction parallel to the 4.0 cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates. (e/m for electron in 1.76×10^{11} C/kg)***

Given: length of plates (D) = 5.0 cm = 5.0×10^{-2} m

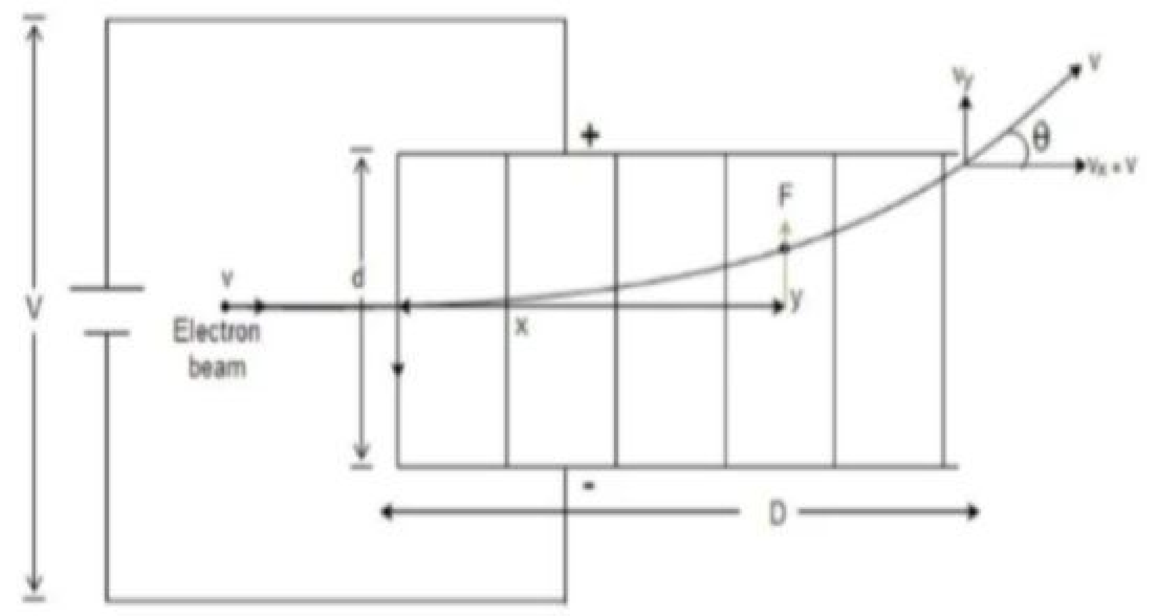
distance between the plates (d) = 3.0 cm =3.0×10^{-2} m

potential supplied between the plates (V) = 400 V

velocity of electron (v) = $1.0 \times 10^7 m/s$

e/m = 1.76×10^{11} C/kg

vertical deflection of electron beam (y) =?



We have,

Vertical deflection of electron beam $(y) = \frac{1}{2}at^2 = \frac{1}{2}(\frac{eE}{m})(\frac{D}{v})^2$

$= \frac{eV}{2md\,v^2}D^2$; here, x = D and E = $\frac{V}{d}$

$y = \frac{1.76\times10^{11} \times400}{2\times3.0\times10^{-2}\times(1.0\times10^7)^2}(5.0 \times 10^{-2})^2 = 3.0 \times 10^{-2}m$

Q.105. ***An electron beam travels in a circular path of radius 4 cm when it enters perpendicular to the direction of uniform magnetic field of 0.01 T. Find the orbital speed, period of revolution and angular frequency of the electron beam.***

Given: radius (r) = 4 cm = 4×10^{-2} m

Magnetic field strength (T) = 0.01 T

Orbital speed (v) =?

Period (T) =?

Angular frequency (ω) =?

Orbital speed of electron (v) = $\frac{eBr}{m} = \frac{1.6\times10^{-19} \times0.01\times4\times10^{-2}}{9.1\times10^{-31}} = 7.0 \times 10^7$ m/s

Period (T) = $\frac{2\pi m}{Be} = \frac{2\pi\times9.1\times10^{-31}}{0.01\times1.6\times10^{-19}} = 3.6 \times 10^{-9}$ s

Angular frequency (ω) = $\frac{2\pi}{T} = \frac{2\pi}{3.6\times10^{-9}}$ =1.7×10^9 rad/s

Q.106. *In Thomson's experiment, the velocity of electrons is 2×10^7 m/s. The magnetic field used is $B=1\times10^{-2}$ T and the radius of the circular path in the magnetic field is r=0.01 m. Calculate the specific charge (e/m) of the electron.*

Given: velocity of electron (v) = 2×10^7 m/s

Magnetic field (T) = 1×10^{-2} T

Radius (r) = 0.01 m

We have, $\frac{e}{m} = \frac{v}{Br} = \frac{2\times 10^7}{1\times10^{-2}\times0.01} = 2\times10^{11}$ C/kg

Photons

Q.107. **The energy required to remove an electron from sodium is 2.3eV. Does the sodium show photoelectric effect for orange light with wavelength 680nm?**

The work function is the minimum energy to just eject or remove the electron then

$$\phi = \frac{ch}{\lambda_0} = \frac{6.625\times10^{-34}\times3\times10^{-8}}{2.3\times1.6\times10^{-19}} = 5397\text{Å}$$

This is the maximum wave length which ejects the electrons. Hence sodium will not show photo electric effect for orange with λ = 680nm.

Q.108. *Calculate the energy of a photon whose (i) frequency is 1000 kHz (ii) wavelength is 5890 Å (iii) wavelength is 1Å. Also express the energy of the photon in eV in each case. Given, 1eV = 1.6×10^{-19} J, h = 6.62×10^{-34} Js and c = 3×10^8 ms^{-1}.*

Solution: Given,

The energy of a photon is given by (E) = $hf = \frac{hc}{\lambda}$ where c is velocity of light.

Planck's constant (h) = 6.625×10^{-34}Js

Velocity of light (c) = 3×10^8 ms^{-1}

We have, 1eV = 1.6×10^{-19} J

(i) Here, frequency (f) = 1000 kHz = 10^6Hz

$\therefore$ Energy (E)= $hf = 6.62\times10^{-34}\times10^6 = 6.62\times10^{-28}$ J.

$$= \frac{6.62\times10^{-28}}{1.6\times10^{-19}} = 4.14\times10^{-9}\text{ eV}$$

(ii) Here, wavelength (λ) = 5890 Å = 5890×10^{-10} m

$$\therefore \text{Energy (E)} = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{5890\times10^{-10}} = 3.37\times10^{-19}\text{J} = \frac{3.37\times10^{-19}}{1.6\times10^{-19}} = 2.11\text{ eV.}$$

(iii) Here, wavelength (λ) = 1Å = 10^{-10} m

$$\therefore \text{Energy (E)} = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{10^{-10}} = \frac{1.99\times10^{-16}}{1.6\times10^{-19}} = 1243.8\text{ eV}$$

Hence, energy is 1243.8eV.

Q.109. ***If 5% of the energy supplied to an incandescent light bulb is radiated as visible light, how many visible light photons are emitted by a 100-watt bulb? Assume the wavelength of all visible photons to be 5600Å. Given h = 6.625 × 10^{-34} J s.***

Solution: Given,

Wavelength (λ) = 5600Å = 5600×10^{-10}m,

Planck's constant (h) = 6.6265×10^{-34} J.s

$$\text{Energy of one photon} = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{5600 \times 10^{-10}} = 3.549 \times 10^{-19}\text{J.}$$

A 100 watt bulb requires 100J of energy per second.

$$\therefore \text{ Energy radiated by the bulb per second} = \frac{100 \times 5}{100} = 5 \text{ J}$$

$$\therefore \text{ Number of photons emitted per second as visible light} = \frac{5}{3.549 \times 10^{-19}} = 1.41 \times 10^{19}$$

Q.110. Calculate the threshold frequency and threshold wavelength of photons which can eject photoelectrons from nickel surface. Given that work function of nickel metal is 5.9 eV and value of Planck's constant 6.62 × 10^{-34} Js.

Solution: Given,

Work function (ϕ) = 5.9 eV = $5.9 \times 1.6 \times 10^{-19}$ J= 9.44×10^{-19}J

If f_0 is threshold frequency, then

$$hf_0 = \phi$$

$$\text{or, } f_0 = \frac{9.44 \times 10^{-19}}{6.62 \times 10^{-34}} = 1.43 \times 10^{15} \text{ Hz}$$

$$\text{The threshold wavelength in given by } (\lambda_0) = \frac{c}{f_0} = \frac{3 \times 10^{8}}{1.43 \times 10^{15}} = 2.098 \times 10^{-7}\text{m}$$

Q.111. ***One mill watt of light of wavelength 4560Å is incident on a cesium surface. Calculate the photoelectric current liberated, assuming a quantum efficiency of 0.5% given (work function ϕ = 1.92eV, h = 6.6 × 10^{-34}Js, C = 3 × 10^{8}m/s***

Solution: Given,

Wavelength (λ) $= 4560\text{Å} = 4560 \times 10^{-10}$m

Power (P) = 1mW $= 10^{-3}$W

Efficiency = 0.5%

Power of the source (P) $= \frac{E}{t} = \frac{nhc}{t\lambda}$

No. of photons emitted per sec $\left(\frac{n}{t}\right) = \frac{P\lambda}{hc}$

$$= \frac{10^{-3} \times (4560 \times 10^{-10})}{6.62 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 2.32 \times 10^{15}$$

Since, the efficiency is 0.5% only 0.5% of the incident photons release the photoelectrons then photoelectric current (I) $= \frac{q}{t}$

$$= \left(\frac{ne}{t}\right) \times 0.5\%$$

$$= \frac{2.32 \times 10^{15} \times 1.6 \times 10^{-19} \times 0.5}{100}$$

$$= 1.856 \times 10^{-6}\text{A}$$

Hence, the photoelectric current is 1.856×10^{-6}A.

Q.112. *A sodium surface ejects photoelectrons when mercury light of frequency 4.7×10^{14} Hz strikes it. The work function of sodium metal is 2.88×10^{-19} J, find the maximum possible kinetic energy of the ejected electrons? Given, $h = 6.62 \times 10^{-34}$ JS.*

Solution: Here,

Work function (ϕ) $= 2.88 \times 10^{-19}$ J

Frequency (f) $= 4.7 \times 10^{14}$ Hz

Planck's constant (h) $= 6.62 \times 10^{-34}$ Js

Let $\frac{1}{2}mv^2_{max}$ be the maximum possible kinetic energy. Then we have

$$hf = \phi + \frac{1}{2}mv^2_{max}$$

or, $\frac{1}{2}mv^2_{max} = hf - \phi$

$$= 6.62 \times 10^{-34} \times 4.7 \times 10^{14} - 2.88 \times 10^{-19}$$

$$= 3.11 \times 10^{-19} - 2.88 \times 10^{-19} = 0.23 \times 10^{-19}\text{J}$$

Hence, the maximum possible kinetic energy of electron is 0.23×10^{-19}J

Q.113. Work function of molybdenum is 5 eV. If ultraviolet light of wavelength 1000 Å falls upon it, find the maximum velocity of the ejected photoelectrons. Planc's constant $h = 6.62 \times 10^{-27}$ ergs, charge $e = 4.8 \times 10^{10}$ e.s.u and $m = 9 \times 10^{-28}$g.

Solution: Given,

Planck's constant (h) $= 6.62 \times 10^{-27}$ ergs

Work function (ϕ) $= hf_0 = 5$ eV $= 5 \times 1.6 \times 10^{-12}$ erg $= 8 \times 10^{-12}$ erg.

Wavelength (λ) $= 1000$ Å $= 1000 \times 10^{-8}$ cm

We have,

$$\text{Energy (E)} = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-27} \times 3 \times 10^{10}}{1000 \times 10^{-8}} = 19.8 \times 10^{-12} \text{ erg.}$$

Kinetic energy of the photoelectrons is given by

$$= \frac{1}{2} mv^2_{max} = hf - hf_0 = 19.8 \times 10^{-12} - 8 \times 10^{-12} \text{ erg}$$

$$\therefore \quad v = \left(\frac{2 \times 11.8 \times 10^{-12}}{m}\right)^{1/2} = \left(\frac{2 \times 11.8 \times 10^{-12}}{9 \times 10^{-28}}\right)^{1/2} = 1.619 \times 10^8 \text{ cm/s}$$

Hence, the velocity of photo electron is 1.619×10^8 cm/s.

Q. 114. ***The work function of potassium, is 2.3 eV. If the photoelectrons are emitted with maximum velocitu of $10^4 ms^{-1}$. calculate freauencu of the incident radiation on the metal. Given that mass of electron, $m = 9.1 \times 10^{-31}$ kg. and Planck's constant $h = 6.62 \times 10^{-34}$ Js.***

Solution: Given,

Work function (ϕ) $= 2.3$ eV $= 2.3 \times 1.6 \times 10^{-19}$J $= 3.68 \times 10^{-19}$ J

Velocity (v) $= 10^4$ ms^{-1}

Planck's constant (h) $= 6.62 \times 10^{-34}$ Js

Mass (m) $= 9.1 \times 10^{-31}$ kg

Then energy of incident photon is given by

$$hf = \phi + \frac{1}{2} mv^2$$

$$= 3.68 \times 10^{-19} + \frac{1}{2} \times 9.1 \times 10^{-31} \times (10^4)^2$$

$$= 3.68 \times 10^{-19} + 4.55 \times 10^{-23}$$

$$= 3.6795 \times 10^{-19} \text{J}$$

$$\text{or, } f = \frac{3.6795 \times 10^{-19}}{6.62 \times 10^{-34}} = 5.56 \times 10^{14} \text{ Hz}$$

Hence, frequency of incident radiation is 5.56×10^{14} Hz.

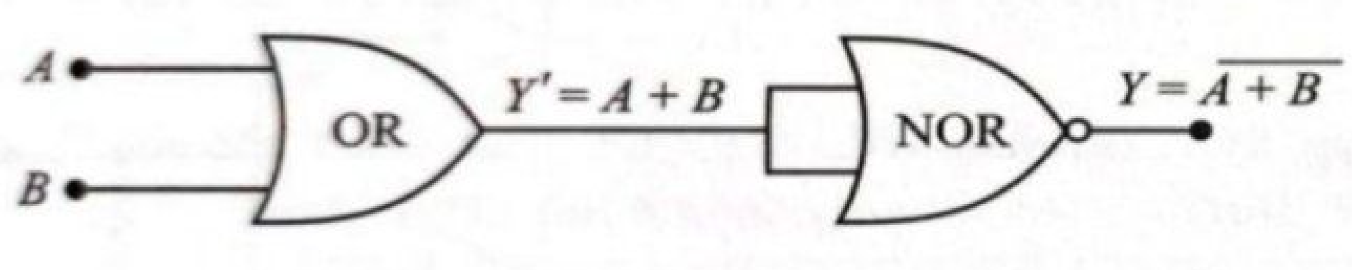
Semiconductor Devices

Q. 115. ***The output of an OR gate is connected to both the inputs of a NOR gate. Draw the logic circuit of this combination of gates and write its truth table.***

Ans. The logic circuit of the combination of two gates is shown is Fig. 28.42. It is clear that :

$$Y' = A + B \text{ and } Y = \overline{(A+B)+(A+B)} = \overline{A+B}$$

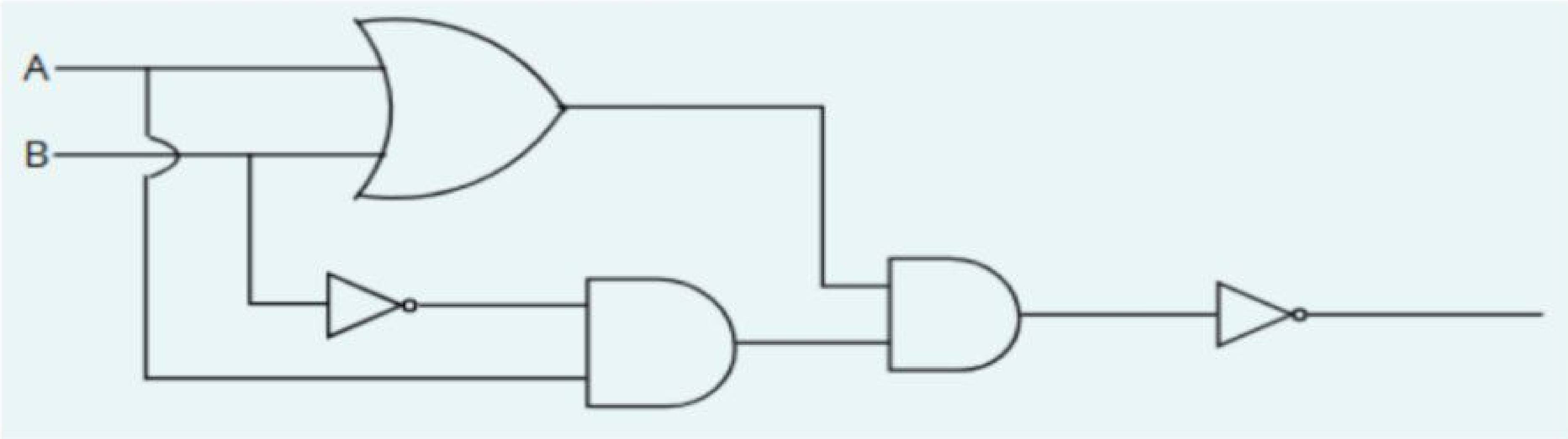
The truth table of the given logic circuit is shown below :



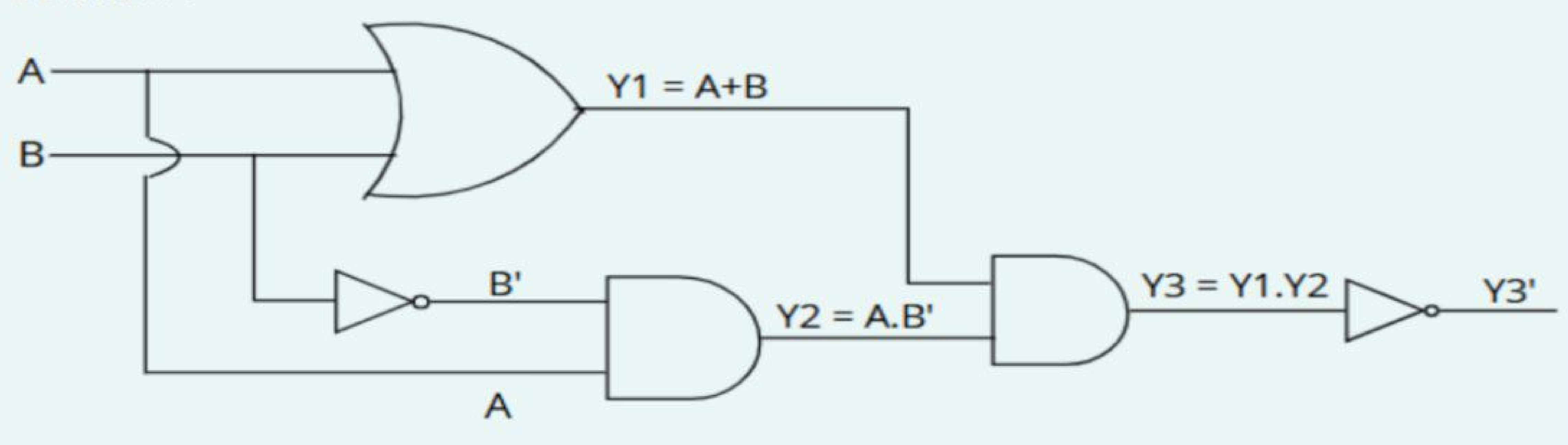
A	B	Y′	Y
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

Fig. 28.42

Q.116. ***Generate the output for given logic circuit.***



Solution ↓



Input		Output				
A	B	B'	Y1 = A+B	Y2 = A.B'	Y3 = Y1.Y2	Y3'
0	0	1	0	0	0	1
0	1	0	1	0	0	1
1	0	1	1	1	1	0
1	1	0	1	0	0	1

Quantisation of Energy

Q.117. ***Calculate the radius of hydrogen atom in its stable state. Also find the velocity of electron in this orbit. Take mass of electron = 9.1 $\times 10^{-31}$ kg, charge on electron = 1.6 $\times 10^{-19}$ C and Planck's constant = 6.62 $\times 10^{-34}$ Js.***

Solution:

Radius of the n^{th} orbit of hydrogen atom is given by

$$r_n = \frac{n^2h^2\varepsilon_o}{\pi e^2 m}$$

Therefore, the radius of stable orbit (n = 1) is given by

Here, h = 6.62×10^{-34} Js, m = 9.1×10^{-31} kg. e = 1.6×10^{-19} C

$$r_1 = \frac{1^2\times(6.625\times10^{-34})^2\times8.85\times10^{-12}}{\pi\times(1.6\times10^{-19})^2\times9.1\times10^{-31}} = 0.53\times10^{-10}\text{m} = 0.53\text{Å}$$

Velocity of electron in this orbit is given by

$$v_1 = \frac{e^2}{2nh\varepsilon_o} = \frac{(1.6\times10^{-19})^2}{2\times1\times6.62\times10^{-34}\times8.85\times10^{-12}} = 2.19 \times 10^6 \text{m/s}$$

Q.118. ***An alpha particle having kinetic energy of 7.68 MeV is projected towards the nucleus of copper (Z = 29). Calculate its distance of nearest approach.***

Solution: Given,

Kinetic energy of alpha particle (KE) = 7.68 MeV= $7.68 \times 1.6 \times 10^{-13}$ J = 1.23×10^{-12} J

At the distance of the nearest approach, we have PE = KE, as all K.E changes to P.E

$$\therefore \quad PE = \frac{K(ze)\,(2e)}{r_0} \quad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore \quad KE = \frac{K(ze)\,(2e)}{r_0} = E_k$$

$$\text{or, } r_0 = \frac{K\,(ze)\,2e}{E_k} = \frac{9\times10^9\times29\times2\times(1.6\times10^{-19})^2}{1.2288\times10^{-12}} = 1.09\times10^{-14}\text{ m.}$$

Hence, the distance of the nearest approach is 1.09×10^{-14} m.

Q.119. *Find the wavelength of the radiation emitted from a hydrogen atom when an electron jumps from third orbit the second orbit. ($\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$, $h = 6.62 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg)*

Energy level of the 3rd orbit, $E_3 = -\frac{me^4}{8\varepsilon_0^2 n^2h^2} = -\frac{me^4}{8\varepsilon_0^2 h^2 \times 9}$

Energy level of the 2nd orbit

$$E_2 = -\frac{me^4}{8\varepsilon_0^2 n^2h^2} = \frac{me^4}{8\varepsilon_0^2 h^2 \times 4}$$

$\therefore$ the frequency of radiation is given by

$$hf = E_3 - E_2 = -\frac{me^4}{8\varepsilon_0^2 h^2 \times 9} - \left(\frac{me^4}{8\varepsilon_0^2 h^2 \times 4}\right)$$

$$\text{or } hf = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore f = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{9-4}{36}\right)$$

$$\therefore \lambda = \frac{c}{f} = \frac{8\varepsilon_0^2 ch^3}{me^4} \times \frac{36}{5}$$

$$\text{or } \lambda = \frac{8 \times (8.85 \times 10^{-12})^2 \times 3 \times 10^8 \times (6.63 \times 10^{-34})^3 \times 36}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4 \times 5} = 7.71 \times 10^{-7}\text{ m}$$

Hence, the wavelength is 7.71×10^{-7} m.

Q.120. *Rydberg constant is equal to $1.09678 \times 10^7 m^{-1}$. Calculate the wavelength of the first number of the Balmer series.*

Solution:

For Balmer series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right), \text{ where } n_2 = 3, 4, 5\ldots\ldots\ldots.$$

For first number of Balmer series,

$$n_2 = 3.$$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\frac{5}{36}$$

$$\text{or } \lambda = \frac{36}{5R}$$

Here, $R = 1.09678 \times 10^7 m^{-1}$

$$\therefore \lambda = \frac{36}{5 \times 1.09678 \times 10^7} = 6.565 \times 10^{-7}\text{m} = 6565 \text{ Å}$$

Q.121. *The first member of Balmer series of hydrogen atom has a wavelength of 6563Å. Compute the wavelength of its second member.*

Solution: Given,

For the first member of Balmer series

$$n_1 = 3, n_2 = 2$$

$$f_1 = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{4} - \frac{1}{9}\right)$$

The corresponding wavelength $\lambda_1 = \frac{c}{f_1}$ ……………(i)

For the second member of the Balmer series

$n_1 = 2, n_2 = 4$

$$f_2 = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{me^4}{8\varepsilon_0^2 h^3}$$

Corresponding wavelength $\lambda_2 = \frac{c}{f_2}$ …………. (ii)

Dividing (ii) by (i)

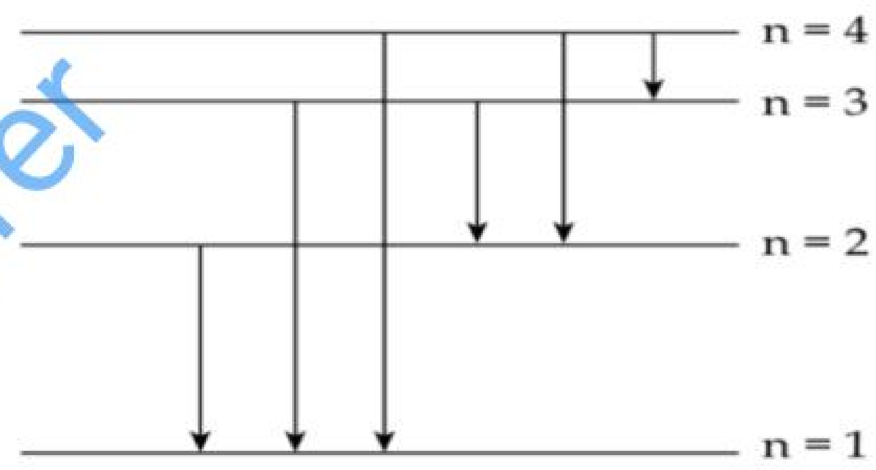
$$\frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2} = \frac{\frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{4} - \frac{1}{9}\right)}{\frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{4} - \frac{1}{16}\right)}$$

$$\text{or, } \frac{\lambda_2}{\lambda_1} = \frac{\frac{9-4}{36}}{\frac{4-1}{16}} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27} \times 6863 = 4861.5 \text{ Å}$$

Hence, the wavelength of second Balmer series is 4861.5 Å

Q.122. ***Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. (Ionization energy for hydrogen atom = 13.6 eV)***



Solution:

Energy of hydrogen atom in the nth state,

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

Energy of the ground state (n = 1) = – Ionization energy

$\therefore \quad E_1 = -13.6$ eV

Energy of the first excited state (n = 2),

$$E_2 = \frac{E_1}{4} = \frac{-13.6}{4} = -3.4\text{eV}.$$

Energy of the 2^{nd} excited state (n = 3),

$$E_3 = \frac{E_1}{9} = \frac{-13.6}{9} = 1.51 \text{ eV}$$

Energy of the 3^{rd} excited (n = 4),

$$E_4 = \frac{E_1}{16} = \frac{-13.6}{16} = -0.85 \text{ eV}$$

$\therefore$ Energy of the incident photon $= \frac{hc}{\lambda}$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}} \text{ J}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV}$$

$= 12.75$ eV

i) When a H - atom in the ground state absorbs this energy, it is excited to a state with energy

= (–13.6 + 12.75) eV

= – 0.85 eV which correspond to n = 4 energy levels.

In general, the number of lines is given by $\frac{n(n-1)}{2}$

For n = 4, the number of lines $= \frac{4(4-1)}{2} = 6$

Hence there are six lines spectrum as shown in energy level diagram.

ii) The longest wavelength line corresponds to smallest energy (λ_m)

i.e. n = 4 to n = 3,

$E_{min} = E_4 - E_3 = -0.85 - (-1.52) = 0.67$ eV

$$\lambda_{max} = \frac{hc}{E_{min}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.67 \times 1.6 \times 10^{-19}} = 18807\text{Å}.$$

Hence, the maximum wavelength is 18807Å.

Q.123. ***A photon and electron have got the same de-Broglie wavelength. Which has greater total energy? Explain.***

For a photon, energy,

$E_1 = hf = hc/\lambda$ .... (i)

For an electron moving with velocity v and effective mass m, de-Broglie wavelength (λ) is given by

$$\lambda = \frac{h}{mv} \quad \text{or} \quad m = \frac{h}{\lambda v} \text{ .... (ii)}$$

Total energy of the electron,

$$E_2 = mc^2 = \frac{hc^2}{\lambda v} \text{ from (ii)}$$

$$\frac{E_2}{E_1} = \frac{hc^2/\lambda v}{hc/\lambda} = \frac{c}{v} > 1 \therefore E_2 > E_1$$

It shows that the total energy of an electron is greater than that of the photon.

Q.124. ***Show that de-Broglie hypothesis of matter wave supports the Bohr's concept of stationary orbit.***

According to de-Broglie hypothesis, the wavelength of the wave associated with electron while moving with velocity v is given by

$$\lambda = \frac{h}{mv} \text{ .... (i)}$$

According to de-Broglie, stationary orbit is that orbit whose circumference is integral multiple of wavelength of wave associated with electron in that orbit.

If λ is the de-Broglie wavelength of electron while revolving in n^{th} orbit of radius r, then

$$2\pi r = n\lambda \quad \text{or } \lambda = 2\pi r/n \text{ ....(ii)}$$

From (i) and (ii)

$$\frac{2\pi r}{n} = \frac{h}{mv} \quad \text{or } mvr = \frac{nh}{2\pi} \text{ i.e. Total angular momentum} = n\left(\frac{h}{2\pi}\right)$$

This is what is stated by Bohr about the stationary orbits.

Q.125. ***The life time of an excited state of an atom is about 10^{-8} sec. Calculate the minimum uncertainty in the determination of the energy of the excited state.***

Solution: Given,

Uncertainty of time (Δt) = 10^{-8}s

Uncertainty in the determination of energy (ΔE) = ?

We have, $\Delta E . \Delta t \geq \frac{h}{2\pi}$

$$\therefore \quad \Delta E \geq \frac{h}{2\pi \Delta t} = \frac{6.6 \times 10^{-34}}{2\pi (10^{-8})} = 1.05 \times 10^{-26} \text{ J} = \frac{1.05 \times 10^{-26}}{1.6 \times 10^{-19} \text{J}} \text{ eV} = 6.56 \times 10^{-8} \text{ eV.}$$

Hence, the minimum uncertainty in the determination of the energy is 6.56×10^{-8} eV.

When the transition takes from E_2 to E_1

$$\lambda_3 = \frac{hc}{E_2 - E_1} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{[-1.5 - (-3.4)] \times 1.6 \times 10^{-19}} = 6.5 \times 10^{-7} \text{ m}$$

Q.126. ***Obtain the de-Broglie wavelength of neutron of kinetic energy 150 eV. [mass of neutron = 1.675×10^{-27}kg]***

Solution: Given,

Kinetic energy (E) $= 150 \text{ eV} = 150 \times 1.6 \times 10^{-19} \text{ J} = 2.4 \times 10^{-17}$ J

Mass of neutron (m) $= 1.675 \times 10^{-27}$ kg.

We have,

$$\text{de-Broglie wavelength } (\lambda) = \frac{h}{mv}$$

$$\text{Also, we have } \frac{1}{2} mv^2 = KE$$

$$\text{or, } mv = \sqrt{2mK.E.}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mK.E.}} = \frac{6.62 \times 10{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 2.4 \times 10^{-17}}} \text{ m} = 2.34 \times 10^{-12} \text{ m}$$

Hence, the de-Broglie wavelength is 2.34×10^{-12} m.

Q.127. ***What is the wavelength of most energetic X-ray emitted when a metal target is bombarded by a electron of energy 100KeV?***

Solution: Given,

Energy of electron = 100 KeV = $100 \times 1.6 \times 10^{-19} \times 10^3$J.

The energy carried by this election is used to produce the X-ray then

$$\text{We have } \frac{1}{2}mv^2 = eV = \frac{hc}{\lambda}$$

$$\text{or, } 100 \times 10^3 \times 1.6 \times 10^{-19} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\text{or, } \lambda = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{100 \times 10^3 \times 1.6 \times 10^{-19}} = 0.124 \times 10^{-10}\text{m}$$

$$\therefore \quad \lambda = 0.124\text{Å}.$$

Hence, the wavelength of most energetic X-ray is 0.124Å .

Q.128. ***Calculate the energy in electron volt of a quantum of X-radiation of wavelength 0.15nm. Given h = 6.62×10^{-34} Js, e = 1.6×10^{-19}C.***

Solution: Given,

Wavelength (λ) $= 0.15\text{nm} = 0.15 \times 10^{-9}$m

Planck's constant (h) $= 6.62 \times 10^{-34}$ Js

Charge of electron (e) $= 1.6 \times 10^{-19}$C

$$\text{Energy of X-radiation} = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.15 \times 10^{-9}} = 1.324 \times 10^{-15}\text{J} = \frac{1.324 \times 10^{-15}}{1.6 \times 10^{-19}} = 8275 \text{ eV}$$

Hence, the energy of x-radiation is 8275 eV.

Q.129. ***Calculate the maximum frequency of the continuous X-rays from an X-rays tube, whose operating voltage is 50,000V. Given h = 6.62 × 10^{-34} J, e = 1.6 × 10^{-19} C and c = 3 × 10^{8}ms^{-1}.***

Solution: Given,

Potential (V) = 50,000 volt

Charge (e) = 1.6 × 10–19C

Planck's constant (h) = 6.62 × 10–34 Js

Maximum frequency of the X-rays is given by

$$f_{max} = \frac{eV}{h}$$

Here,

$$\therefore \quad f_{max} = \frac{1.6 \times 10^{-19} \times 50000}{6.62 \times 10^{-34}} = 1.21 \times 10^{19} \text{ Hz}$$

Hence, the maximum frequency is 1.21 × 10^{19} Hz.

Q.130. ***The anode to cathode potential applied in an X-ray tube is 20,000V. Calculate (i) the resultant energy of the electrons and (ii) the minimum wavelength of the X-rays produced. Given, h = 6.62 × 10^{-34}Js and e = 1.6 × 10^{-19}C.***

Solution: Given,

Potential (V) = 20,000 V

Planck's constant (h) = 6.62 × 10^{-34} Js

Electronic charge (e) = 1.6 × 10^{-19}C.

Energy of the electrons = eV = 20000 × 1.6 × 10^{-19}J= 3.2 × 10^{-15} J

If λ_{min} is the minimum wavelength of X-rays produced, then

$$\frac{hC}{\lambda_{min}} = eV$$

$$\text{or,} \quad \lambda_{min} = \frac{hC}{eV} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{3.2 \times 10^{-15}} = 6.21 \times 10^{-11}\text{m}$$

Hence, the minimum wavelength of X-rays is 6.21 × 10^{-11}m.

Q.131. ***An X-ray tube has the anode potential equal to 15kV. Current is found to be of the strength 20 mA. How many electrons hit the target per second and how much heat is generated at the anode per second? Given e = 1.6 × 10^{-19}C.***

Solution: Given,

Potential (V) = 15kV = 15000 V

Electronic charge (e) = 1.6 × 10^{-19}C

Current (I) = 20mA = 20 × 10^{-3}A

Let n be the number of electrons hitting the target per second,

Then, I = ne

$$\text{or,} \quad n = \frac{I}{e} = \frac{20 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$$= 12.5 \times 10^{16}$$

$$= 125 \times 10^{15} \text{ no. of electrons/sec.}$$

Also, the maximum heat energy generated at the anode per sec = I × V = 20 × 10^{-3} × 1500 = 300W

Q.132. ***The lattice constant of a crystal is 10^{-7}cm. What is the upper limit for the wavelength of X-rays with this crystal can be studied?***

Solution:

Given,

Lattice constant (d) = 10^{-7}cm = 10^{-9}m

We have, $2d\sin\theta = n\lambda$

For maximum value $(\sin\theta)_{max} = 1$, n = 1

$\therefore\ 2d = \lambda$

$\lambda = 2d$

$= 2 \times 10^{-9} = 20 \times 10^{-10}\text{m} = 20\text{Å}$

Hence, the wavelength (λ) = 20Å.

Q.133. ***The spacing of atomic planes in a crystal is 1.1×10^{-10}m and then a monochromatic beam of X-ray is incident on them of glancing angle of 50° a first order image is produced calculate the wavelength what is the glancing angle for the second order image?***

Solution: Given,

Case I

Space between atomic planes (d) = 1.1×10^{-10}m

Glance angle (θ_1) = 5^o

Order of image produced (n) = 1

Wavelength (λ) = ?

We have,

$2d\sin\theta_1 = n_1\lambda$

or $\lambda = \frac{2d\sin\theta_1}{n} = \frac{2 \times 1.1 \times 10^{-10} \times \sin5°}{1}$

$\therefore\ \lambda = 1.92 \times 10^{-11}$m

Cass II:

Space between atomic planes (d) = 1.1×10^{-10}m

Glancing angle (θ_2) = ?

Order of image produced (n_2) = 2

Wavelength (λ) = 1.92×10^{-11}m

We have,

$2d\sin\theta_2 = n_2\lambda$

or $\sin\theta_2 = \frac{n_2}{2d}\lambda = \frac{2\times1.92\times10^{-11}}{2\times1.1\times10^{-10}}$

$\theta_2 = 10^o$

Hence, the required wavelength is 1.92×10^{-11}m and glancing angle is 10^0

Q.134. ***At a certain instant a piece of radioactive material contains 10^{12} atoms. The half-life of the material is 15 days. Calculate the number of disintegrations per second.***

Solution: Given,

Half-life ($t_{1/2}$) = 15 days = 15 × 24 × 60 × 60s
Number of atoms present at any instant = 10^{12}

Now, $t_{1/2} = \frac{0.6931}{\lambda}$

We have, $\frac{dN}{dt} = \lambda N = \frac{0.6931}{15 \times 24 \times 60 \times 60} \times 10^{12} = 5.347 \times 10^5$ dis/s

Hence, the number of disintegrations per second is 5.347×10^5.

Q.135. ***Calculate the mass in grams of a radioactive sample Pb-214 having an activity of 3.7×10^4 decays/s and a half-life of 26.8 minutes. Avogadro number = 6.02×10^{24} mole.***

Solution: Given,

Activity $\left(\frac{dN}{dt}\right) = 3.7 \times 10^4$ dis/sec   Half-life ($t_{1/2}$) = 26.8 min = 1608 sec

Avogadro number $N_A = 6.02 \times 10^{24}$ atoms/mole   Mass of sample (m) = ?

We have,

From radioactive decay law

$\left(\frac{dN}{dt}\right) = N\lambda$ .... (i)

Also we have,

$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1608} = 4.3 \times 10^{-4}$/s

From (i)

$3.7 \times 10^4 = N \times 4.3 \times 10^{-4}$

$N = \frac{3.7}{4.3} \times 10^8$ no. of atoms

$N = 8.6 \times 10^7$ no. of atoms

According to question

214g of Pb contains 6.02×10^{24} no. of atoms

1 atom = $\frac{214}{6.02 \times 10^{23}}$gm

8.6×10^7 atoms = $\frac{214}{6.02 \times 10^{23}} \times 8.6 \times 10^7 = 5.07 \times 10^{-15}$gm

Hence, mass of sample is 5.07×10^{-15}gm.

Q.136. ***After a series of alpha and beta decays, plutonium 246 ($_{94}Pu^{249}$) becomes lead 207 ($_{82}Pu^{207}$). How may alpha and beta particles are emitted in the complete decay scheme? State the reason in support of your answer.***

Solution: Given,

Let, there are m and n number of α – β particles are emitted. Then the reaction can be written as

$$_{94}Pu^{249} = {}_{82}Pu^{207} + m({}_2He^4) + n({}_{-1}e^0)$$

Now, 249 = 207 + 4m + 0 [mass and atomic number should be conserved]

$4m = 32$

$\therefore \quad m = 8$

Similarly, 94 = 82 + 2m – n

$2m - 12 = n$

$n = 4$

Hence, the 4-β particles are emitted.

Q.137. ***Calculate the time required for 1% of a sample of radium of disintegrate. Assume the half-life of radium to be 1500 years.***

Solution: If 1% of the sample disintegrates. Then,

$$\frac{N}{N_0} = \frac{99}{100}$$

Half life ($t_{1/2}$) = 1500 years

$$\text{or,} \quad e^{-\lambda t} = \frac{99}{100} \quad \left(\frac{N}{N_0} = e^{-\lambda t}\right)$$

$$\text{or} \quad e^{\lambda t} = \frac{100}{99}$$

$$\text{or} \quad \lambda t = \ln \frac{100}{99}$$

$$\text{or,} \quad \frac{0.693}{t_{1/2}} t = \ln\left(\frac{100}{99}\right)$$

$$\text{or} \quad t = \frac{t_{1/2}}{0.693} \times \ln \frac{100}{99} = \frac{1500}{0.693} \times \ln \frac{100}{99} = 21.75 \text{ years,}$$

Hence, the time required is 21.75 years.

Q.138. ***A radioactive source has decayed to $\left(\frac{1}{128}\right)^{th}$ of its initial activity after 50 days. What is its half life?***

Solution: Activity $\left(\frac{dN}{dt}\right) \propto N$

$$\frac{N}{N_0} = \frac{1}{128}$$

$$\text{But,} \quad \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \quad e^{-\lambda t} = \frac{1}{128} \text{ or } e^{\lambda t} = 128$$

$$\text{or,} \quad \lambda t = \ln (128)$$

$$\text{or,} \quad \lambda = \frac{1}{t} \ln (128) = \frac{1}{50} \ln (128)$$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693 \times 50}{\ln (128)} = 7.12 \text{ days}$$

<u>Alternative Method</u>

$$\frac{N}{N_0} = \frac{1}{128}$$

$$\text{or,} \quad \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} = \left(\frac{1}{2}\right)^7$$

$$\text{or,} \quad \frac{50}{t_{1/2}} = 7$$

$$\therefore \quad t_{1/2} = 7.12\text{days}$$

Hence, the half life is 7.12days.

Q.139. ***The initial number of atoms in a radioactive element is 6.0 × 10^{20} and its half life is 10 hr. Calculate (a) the number of atoms which have decayed in 30hr (b) the amount of energy liberated if the energy liberated per atom decay is 4.0 × 10^{-15}J.***

Solution: Given,

Initial number of atoms (N_0) = 6.0×10^{20} Half life ($t_{1/2}$) = 10 hrs.

Time (t) = 30 hrs. No. of atoms present (N) =?

We know, $N = N_0 e^{-\lambda t} = N_0 e \frac{-\ln 2}{t_{1/2}} t$

$= 6.0 \times 10^{20} \times e - \frac{0.693 \times 30}{10}$

$= 6.010^{20} \times e^{-2.079}$

$= \frac{6.0 \times 10^{20}}{e^{2.079}} = \frac{6.0 \times 10^{20}}{7.996} = \frac{6.0 \times 10^{20}}{8} = 0.75 \times 10^{20}$

∴ Number of atoms decayed.

$= N_0 - N = 6.0 \times 10^{20} - 0.75 \times 10^{20} = 5.25 \times 10^{20}$

∴ Amount of energy liberated (E) = $5.25 \times 10^{20} \times 4.0 \times 10^{-13} = 2.1 \times 10^{8}$ Joules

Hence, the amount of energy liberated is 2.1×10^{8} Joules.